Beyond Quantum Mechanics

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11th Mathematical Physics Meeting
Beograd, 26-30 August, 2024

BUFFALO
University at Buffalo

Based on:

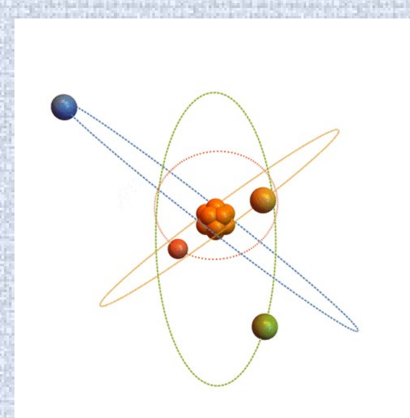
"Beyond Quantum Mechanics"
Sam Powers, Dejan Stojkovic
Eur.Phys.J.C 82 (2022) 690

"Testing a discrete model for quantum spin with two sequential Stern-Gerlach detectors and photon Fock states"
S. Powers, G. Xu, H. Fotso, T. Thomay, D. Stojkovic
e-Print: 2304.13535 [quant-ph]

Outline

- Binary sequences
- Relations between the sequences
- Measures – quantum numbers
- Probabilities – counting sequences
- Clebsch-Gordan coefficients
- Wigner’s formula
- Single photon state experiments
- *Summary*

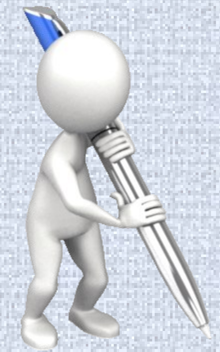
Current state-of-the-art in QM



QM is one of the cornerstones of modern physics

Many pressing questions:

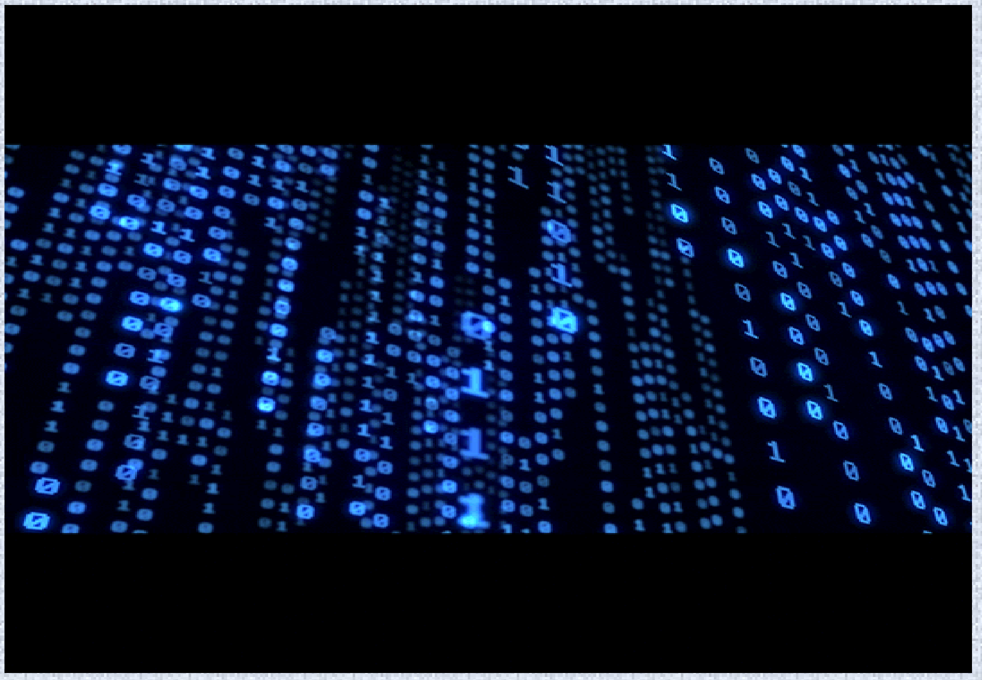
- Collapse of the wave function
- The status of an observer
- Born rule (why is the wavefunction squared probability?)
- Determinism vs non-determinism
- Quantum theory of spacetime



In almost 100 years we have not made any satisfactory progress

We need something radically different!

Quantum mechanics from information



Statement

Humans discovered quantum mechanics when they discovered counting

(*though perhaps they were not aware of it*)

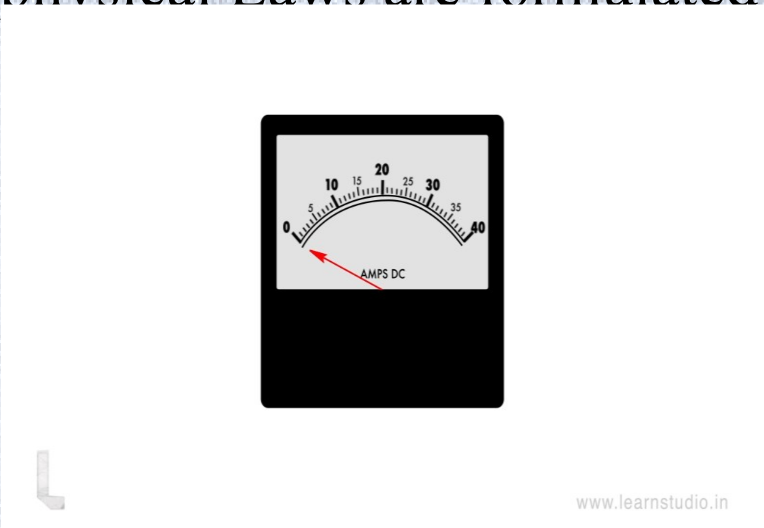
Tension

We learn about our world by doing measurements

Every conceivable measurement can be reduced to counting

however

Our physical Laws are formulated as differential equations (continuity)



$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon} \quad \text{(Gauss' Law)}$$

$$\nabla \cdot \mathbf{H} = 0 \quad \text{(Gauss' Law for Magnetism)}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \quad \text{(Faraday's Law)}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \quad \text{(Ampere's Law)}$$

This is not simply a matter of improving precision or collecting more data

It is a fundamental difference between our experience of the physical world and the theories we use to model those experiences

Information

There is very little or no content/information if you have only one element

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

You need at least two basis elements to convey meaningful information

Say 0 and 1

Binary sequences

Fundamental objects are base-2 (binary) sequences of length n

$$\begin{pmatrix}1\\0\\1\\1\\0\\1\end{pmatrix} = \begin{pmatrix}True\\False\\True\\True\\False\\True\end{pmatrix} = \begin{pmatrix}+\\-\\+\\+\\-\\+\end{pmatrix} = \begin{pmatrix}Yang\\Yin\\Yang\\Yang\\Yin\\Yang\end{pmatrix} = \cdots$$

The most irreducible way to encode information

Label all such sequences with $S^1(n)$:

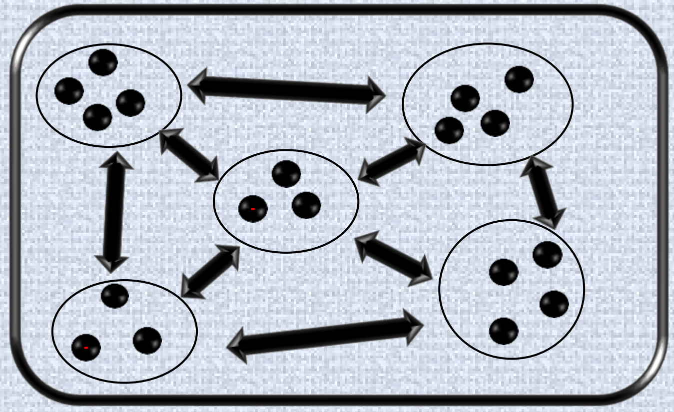
$$\begin{pmatrix}0\\0\\0\\0\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\\0\\0\end{pmatrix}, \dots, \begin{pmatrix}1\\1\\0\\0\\0\\1\end{pmatrix}, \dots, \begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix} \quad \in \ \mathbf{S^1(6)}$$

Lecture from quantum statistical mechanics

D. N. Page, *Phys. Rev. Lett. 71, 1291 (1993)*

- If we divide a system into small subsystems

- Very little information is in the small subsystems



- All the information is in the correlations between the systems

(alphabet has only 26 symbols, correlations between the letters crucial)

Relationships between the sequences

Set S^2 (n) is the set of all relationships between two sequences

$$\begin{pmatrix}1\\0\\0\\1\\0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\0\\1\\1\\0\\0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\\0 & 1\\1 & 1\\0 & 0\\1 & 0\end{pmatrix} \equiv \begin{pmatrix}C\\A\\D\\B\\A\\C\end{pmatrix}$$

$s^1 \in S^1$ $\quad s'^1 \in S^1$ $\quad s^1 \otimes s'^1 \in S^2$

Define: 0 0 ≡ A 1 1 ≡ B 1 0 ≡ C 0 1 ≡ D

Set S^2 (n) is therefore comprised of base-4 sequences

Relationships between the sequences

Set S^3 (n) is the set of all relationships between three sequences

$$\begin{pmatrix}1\\0\\0\\1\\0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\0\\1\\1\\0\\0\end{pmatrix} \otimes \begin{pmatrix}0\\0\\1\\1\\0\\1\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 1 & 1\\1 & 1 & 1\\0 & 0 & 0\\1 & 0 & 1\end{pmatrix}$$

$$s^1 \otimes s'^1 \otimes s''^1 \in S^3$$

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Basis

Set S^3 (n) is made of base-8 sequences

We could keep going – S^4 (n) , S^5 (n) …

Counts and measures

- Set $S^2(n)$ is the set of all relationships between two sequences

$$\begin{pmatrix}1\\0\\0\\1\\0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\0\\1\\1\\0\\0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\\0 & 1\\1 & 1\\0 & 0\\1 & 0\end{pmatrix} \equiv \begin{pmatrix}C\\A\\D\\B\\A\\C\end{pmatrix} \qquad \boldsymbol{\tilde{A} = 2, \tilde{B} = 1, \tilde{C} = 2, \tilde{D} = 1}$$

- Number of times a particular base-4 basis element appears is a count.

- Label the counts as $\boldsymbol{\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}}$

- Obviously $\boldsymbol{\tilde{A} + \tilde{B} + \tilde{C} + \tilde{D} = n}$

Measures

0 0 ≡ A   1 1 ≡ B   1 0 ≡ C   0 1 ≡ D

$$(1) \quad \boldsymbol{j = \frac{\tilde{C}+\tilde{D}}{2}} \qquad (2) \quad \boldsymbol{m = \frac{\tilde{C}-\tilde{D}}{2}}$$

$$(3) \quad \boldsymbol{g = \frac{\tilde{A}+\tilde{B}}{2}} \qquad (4) \quad \boldsymbol{l = \frac{\tilde{A}-\tilde{B}}{2}}$$

with $\quad -j \le m \le j \qquad -g \le l \le g$

$$j = 2 \;\Rightarrow\; \tilde{C} + \tilde{D} = 4 \;\Rightarrow\; \begin{array}{l} \tilde{C} = 4, \tilde{D} = 0 \\ \tilde{C} = 3, \tilde{D} = 1 \\ \tilde{C} = 2, \tilde{D} = 2 \\ \tilde{C} = 1, \tilde{D} = 3 \\ \tilde{C} = 0, \tilde{D} = 4 \end{array} \;\Rightarrow\; \tilde{C} - \tilde{D} = 4, 2, 0, -2, -4$$

$$-2 \le m \le 2$$

Measures *j* and *m*: *quantum numbers*

(1) $\boldsymbol{j} = \frac{\tilde{\boldsymbol{C}}+\tilde{\boldsymbol{D}}}{\mathbf{2}}$ (2) $\boldsymbol{m} = \frac{\tilde{\boldsymbol{C}}-\tilde{\boldsymbol{D}}}{\mathbf{2}}$

with $-j \leq m \leq j$

If counts $\tilde{\boldsymbol{C}}$ and $\tilde{\boldsymbol{D}}$ are given in units of $\hbar$ ➡

j looks like the total angular momentum quantum number
m looks like the z-component of the angular momentum

Measures label all sequences

- Label sequences with numbers *j,m,g,l* ➡ $s^2_{j,m,g,l} \in S^2(j, m, g, l)$

- More than one sequence with the same numbers

$$s^2_{3/2,1/2,3/2,1/2} = \begin{pmatrix} C \\ A \\ D \\ B \\ A \\ C \end{pmatrix} \qquad s^2_{3/2,1/2,3/2,1/2} = \begin{pmatrix} A \\ C \\ B \\ D \\ A \\ C \end{pmatrix}$$

$$j = \frac{\tilde{C}+\tilde{D}}{2} \qquad m = \frac{\tilde{C}-\tilde{D}}{2}$$

$$g = \frac{\tilde{A}+\tilde{B}}{2} \qquad l = \frac{\tilde{A}-\tilde{B}}{2}$$

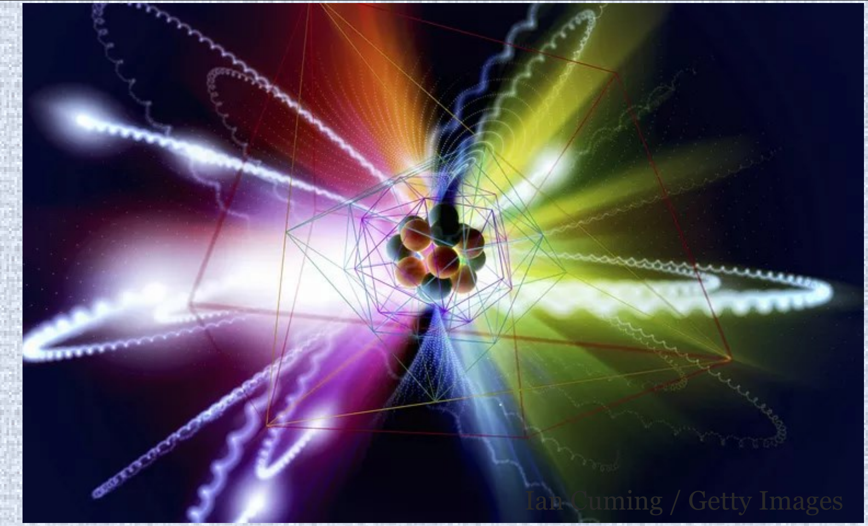
- Degeneracy is determined by permutations of elements

j,m,g,l is a complete set of measures (up to permutations)

Elementary particles: states labeled with *j* and *m*

$$s^2_{j,m,g,l} \in S^2(j, m, g, l)$$

with $-j \leq m \leq j$



- Elementary particles are states labeled with ***j*** and ***m***
- Particles are not fundamental objects

Particles are correlations between the sequences!

Referent sequence

Look again at the product of two sequences

$$\begin{pmatrix}1\\0\\0\\1\\0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\0\\1\\1\\0\\0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\\0 & 1\\1 & 1\\0 & 0\\1 & 0\end{pmatrix} \equiv \begin{pmatrix}C\\A\\D\\B\\A\\C\end{pmatrix} \Rightarrow s^2_{\frac{3}{2},\frac{1}{2},\frac{3}{2},\frac{1}{2}}$$

$$s^1{}_0 \otimes s^1_{j,m,g,l} = s^2_{j,m,g,l}$$

Call the $s^1{}_0$ sequence on the left – **the referent sequence**

If we keep the referent sequence fixed, we can assign *j,m,g,l* to all S^1 sequences

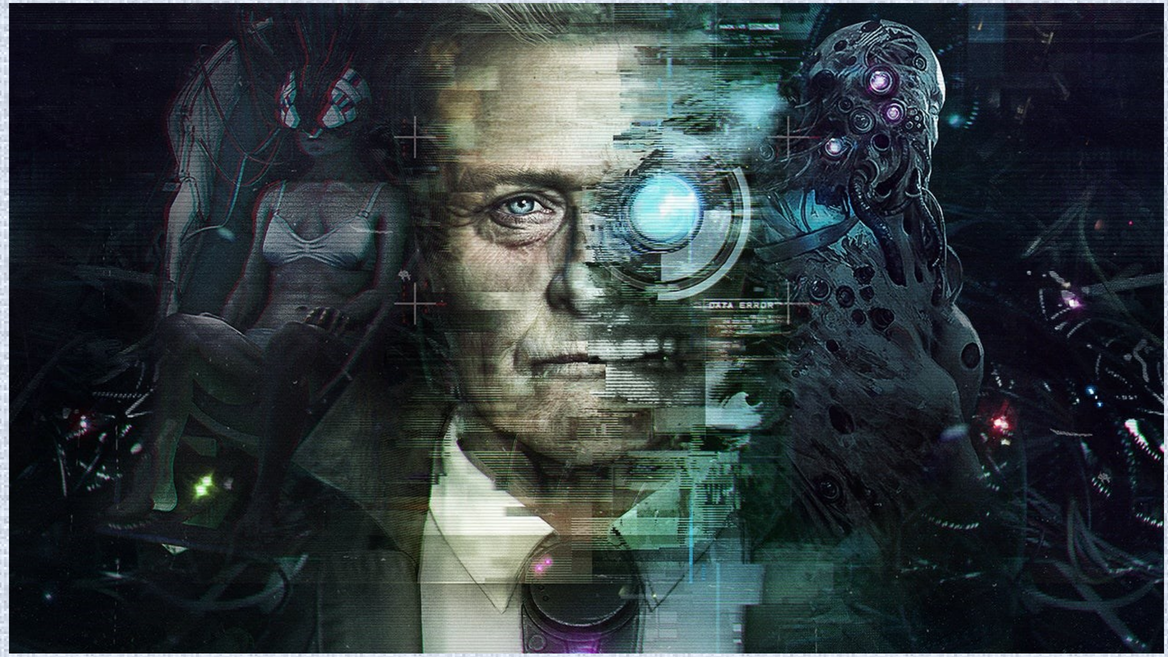
Sequence on the left looks at the sequence to the right and sees measures *j,m,g,l*

The orientation is important due to the asymmetry of C = 10 and D = 01 basis

$$s^1{}_0 \otimes s^1_{j,m,g,l} \rightarrow s^{*1}_{j,-m,g,l} \otimes s^1{}_0$$

Referent sequence

Referent sequence plays the role of an observer, or vacuum
(everything is defined with respect to it)



Observer is an integral part of the system!

Referent sequence

When the referent sequence looks at itself it cannot see *C* and *D*, so ***j=m=0***

$$\begin{pmatrix}1\\0\\0\\1\\0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\1\\0\\1\end{pmatrix} = \begin{pmatrix}1 & 1\\0 & 0\\0 & 0\\1 & 1\\0 & 0\\1 & 1\end{pmatrix} \equiv \begin{pmatrix}B\\A\\A\\B\\A\\B\end{pmatrix} \longrightarrow s^2_{0,0,3,0}$$

$$s^1{}_0 \otimes s^1_0 = s^2_{0,0,g,l}$$

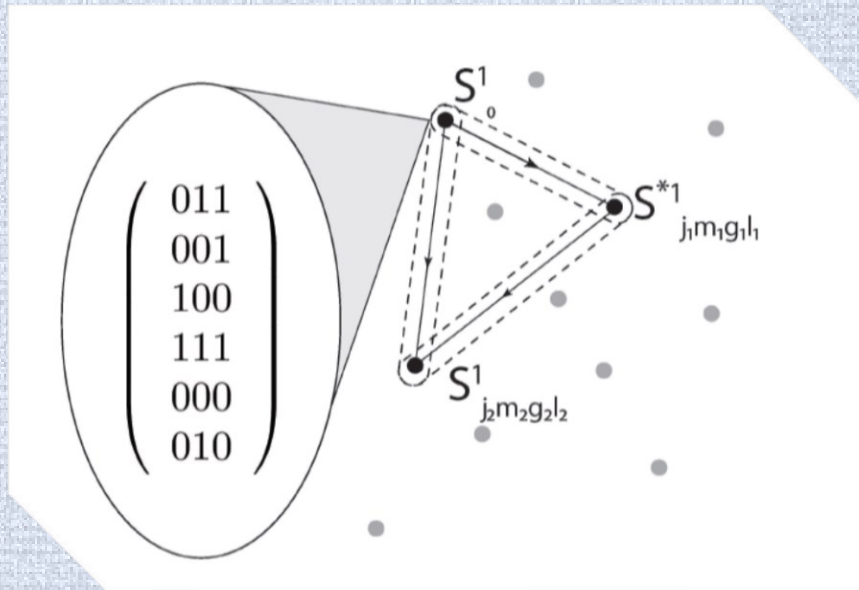
Vacuum must be a scalar (does not carry any angular momentum)!

QM rules for angular momentum addition

What happens when $s^1_{j_1,m_1,g_1,l_1}$ looks at $s^1_{j_2,m_2,g_2,l_2}$?

$$s^{*1}_{j_1,m_1,g_1,l_1} \otimes s^1{}_0 \otimes s^1_{j_2,m_2,g_2,l_2} \in S^3$$

Place the reference sequence $s^1{}_0$ in the middle



Directed graph: three vertices and three directed edges

$$s^{*1}_{j_1,m_1,g_1,l_1} \otimes s^1{}_0 \otimes s^1_{j_2,m_2,g_2,l_2} = s^3{}_{J,M,G,L}$$

QM rules for angular momentum addition

Two sequences

$$s^{1}{}_{0} \otimes s^{1}_{j,m,g,l} = s^{2}_{j,m,g,l}$$

0 0 ≡ A 1 1 ≡ B 1 0 ≡ C 0 1 ≡ D

Three sequences

$$\boldsymbol{s^{*1}_{j_1,m_1,g_1,l_1} \otimes s^{1}{}_{0} \otimes s^{1}_{j_2,m_2,g_2,l_2} = s^{3}{}_{J,M,G,L}}$$

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Basis

Define:

$$J = \frac{1}{2}(1\widetilde{00} + 1\widetilde{10} + 0\widetilde{01} + 0\widetilde{11}) \longleftrightarrow j = \frac{\widetilde{C} + \widetilde{D}}{2}$$

$$M = \frac{1}{2}(1\widetilde{00} + 1\widetilde{10} - 0\widetilde{01} - 0\widetilde{11}) \longleftrightarrow m = \frac{\widetilde{C} - \widetilde{D}}{2}$$

$$G = \frac{1}{2}(0\widetilde{00} + 0\widetilde{10} + 1\widetilde{11} + 1\widetilde{01}) \longleftrightarrow g = \frac{\widetilde{A} + \widetilde{B}}{2}$$

$$L = \frac{1}{2}(0\widetilde{00} + 0\widetilde{10} - 1\widetilde{11} - 1\widetilde{01}) \longleftrightarrow l = \frac{\widetilde{A} - \widetilde{B}}{2}$$

Relationships between measures

$$s^{*1}_{j_1,m_1,g_1,l_1} \otimes s^1{}_0 \otimes s^1_{j_2,m_2,g_2,l_2} = s^3{}_{J,M,G,L}$$

$$\updownarrow$$

$$J = \frac{1}{2}(1\widetilde{00} + 1\widetilde{10} + 0\widetilde{01} + 0\widetilde{11})$$

For example $s^{*1}_{1,0,1,-1} \otimes s^1{}_0 \otimes s^1_{\frac{1}{2},+\frac{1}{2},\frac{3}{2},-\frac{1}{2}} = s^3{}_{J,M,G,L}$

$$j_1 = 1 \rightarrow \frac{1}{2}(\tilde{C}_1 + \tilde{D}_1) = 1 \qquad j_2 = \frac{1}{2} \rightarrow \frac{1}{2}(\tilde{C}_2 + \tilde{D}_2) = \frac{1}{2}$$

One possibility

$$\begin{pmatrix} 1 & 1 & [0] \\ 1 & 1 & 1 \\ [1] & 0 & 0 \\ [0] & 1 & 1 \end{pmatrix}$$

[ ] is element that contributes
- they **do not** overlap

$$J = j_1 + j_2 = \frac{3}{2}$$

Another possibility

$$\begin{pmatrix} [0] & 1 & [0] \\ 1 & 1 & 1 \\ [1] & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

[ ] is element that contributes
- they **do** overlap

$$J = j_1 + j_2 - 1 = \frac{1}{2}$$

In general

$$|j_1 - j_2| \geq J \leq j_1 + j_2$$

Non-determinism

$$|j_1 - j_2| \geq J \leq j_1 + j_2$$

This is a non-deterministic relation (typical for quantum mechanics)

Non-determinism is a direct consequence of obscuring information about the exact composition of a sequence!

Relationships between measures

$s^{1}{}_{0} \otimes s^{1}_{j,m,g,l} = s^{2}_{j,m,g,l}$ **0 0 ≡ A 1 1 ≡ B 1 0 ≡ C 0 1 ≡ D**

The reference sequence contributes a 0 basis element to the A and D basis elements

While it contributes a 1 basis element to both the B and C basis elements.

Number of zeros in the referent sequence is $\tilde{0}_0$ $\tilde{A} + \tilde{D} = \tilde{0}_0$

Number of ones in the referent sequence is $\tilde{1}_0$ $\tilde{B} + \tilde{C} = \tilde{1}_0$

$$\boldsymbol{s^{*1}_{j_1,m_1,g_1,l_1} \otimes s^{1}{}_{0} \otimes s^{1}_{j_2,m_2,g_2,l_2} = s^{3}{}_{J,M,G,L}}$$

Use $\boldsymbol{m} = \dfrac{\boldsymbol{\tilde{C} - \tilde{D}}}{\boldsymbol{2}}$

And get $\mathbf{M} = \boldsymbol{m_1 + m_2}$

Conservation law (*not imposed by hand*)!

Ontic vs Epistemic states

- **Epistemic states are observed in experiments (by us humans)**
- **Ontic states are underlying reality (not possible to observe)**

Epistemic state of a particle is labeled by *(j,m)*
Ontic states are ensembles of base-4 sequences labeled by *j,m,g,l*

A state with $j=1/2$ (e.g. electron) is described by the set $s^2_{j=\frac{1}{2},m,g,l}$

$$s^2_{j=\frac{1}{2},m,g,l} = \alpha\, s^2_{j=\frac{1}{2},m=+\frac{1}{2},g,l} + \beta s^2_{j=\frac{1}{2},m=-\frac{1}{2},g,l}$$

Electron wavefunction $|\boldsymbol{\psi}\rangle = \boldsymbol{\alpha}|\uparrow\rangle + \boldsymbol{\beta}|\downarrow\rangle$

The more sequences we have with given *j* and *m*,
the greater probability to obtain this state in a given process

Counting function

Define the counting function $\Phi(j,m,g,l) = \frac{n!}{\widetilde{\boldsymbol{A}}!\widetilde{\boldsymbol{B}}!\widetilde{\boldsymbol{C}}!\widetilde{\boldsymbol{D}}!}$

permutations with repetition

Assign $\Phi(j,m,g,l)$ to each sequence with given j,m,g,l

Counting function counts all the distinct sequences with the same j,m,g,l

$$s^2_{3/2,1/2,3/2,1/2} = \begin{pmatrix} C \\ A \\ D \\ B \\ A \\ C \end{pmatrix} \qquad \Phi(3/2,1/2,3/2,1/2) = \frac{6!}{\widetilde{\mathbf{2}}!\widetilde{\mathbf{1}}!\widetilde{\mathbf{2}}!\widetilde{\mathbf{1}}!} = 180$$

Clebsch-Gordan coefficients

CG coefficients are densely packed with very important physics

Unfortunately, this is not so obvious within the formalism of QM

Explicit direct sum decomposition of the tensor product of two irreducible representations into irreducible representations, in cases where the numbers and types of irreducible components are already known abstractly. **Wikipedia**

$$\left|\frac{1}{2},\frac{1}{2},0,0\right\rangle = -\sqrt{\frac{1}{2}}\left|\frac{1}{2},\frac{1}{2},\frac{-1}{2},\frac{1}{2}\right\rangle + \sqrt{\frac{1}{2}}\left|\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{-1}{2}\right\rangle$$

$$\left|\frac{1}{2},\frac{1}{2},1,1\right\rangle = \left|\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right\rangle$$

$$\left|\frac{1}{2},\frac{1}{2},1,0\right\rangle = \sqrt{\frac{1}{2}}\left|\frac{1}{2},\frac{1}{2},\frac{-1}{2},\frac{1}{2}\right\rangle + \sqrt{\frac{1}{2}}\left|\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{-1}{2}\right\rangle$$

$$\left|3,\frac{3}{2},\frac{3}{2},\frac{-1}{2}\right\rangle = -\sqrt{\frac{2}{7}}\left|3,\frac{3}{2},-2,\frac{3}{2}\right\rangle + \sqrt{\frac{12}{35}}\left|3,\frac{3}{2},-1,\frac{1}{2}\right\rangle - \sqrt{\frac{9}{35}}\left|3,\frac{3}{2},0,\frac{-1}{2}\right\rangle + \sqrt{\frac{4}{35}}\left|3,\frac{3}{2},1,\frac{-3}{2}\right\rangle$$

$$\left|3,\frac{3}{2},\frac{3}{2},\frac{3}{2}\right\rangle = -\sqrt{\frac{1}{35}}\left|3,\frac{3}{2},0,\frac{3}{2}\right\rangle + \sqrt{\frac{4}{35}}\left|3,\frac{3}{2},1,\frac{1}{2}\right\rangle - \sqrt{\frac{2}{7}}\left|3,\frac{3}{2},2,\frac{-1}{2}\right\rangle + \sqrt{\frac{4}{7}}\left|3,\frac{3}{2},3,\frac{-3}{2}\right\rangle$$

$$\left|3,\frac{3}{2},\frac{9}{2},\frac{5}{2}\right\rangle = \sqrt{\frac{5}{12}}\left|3,\frac{3}{2},1,\frac{3}{2}\right\rangle + \sqrt{\frac{1}{2}}\left|3,\frac{3}{2},2,\frac{1}{2}\right\rangle + \sqrt{\frac{1}{12}}\left|3,\frac{3}{2},3,\frac{-1}{2}\right\rangle$$

$$\left|3,\frac{3}{2},\frac{9}{2},\frac{9}{2}\right\rangle = \left|3,\frac{3}{2},3,\frac{3}{2}\right\rangle$$

Clebsch-Gordan coefficients

When adding two angular momenta, $\boldsymbol{J = j_1 + j_2, j_1 + j_2 - 1, \ldots, |j_1 - j_2|}$

Probability distribution is given by the Clebsch-Gordan coefficients

$$|JM\rangle = \sum_{m_1=-j_1}^{j_1} \sum_{m_2=-j_2}^{j_2} |j_1 m_1 j_2 m_2\rangle \underbrace{\langle j_1 m_1 j_2 m_2 | JM\rangle}_{\text{Clebsch-Gordan coefficients}}$$

e.g. $j_1 = j_2 = 1$ one possibility is $J = 2, M = 0$

$$|2\ 0\rangle = \frac{1}{\sqrt{6}}|1\ 1\ 1 - 1\rangle + \frac{2}{\sqrt{6}}|1\ 0\ 10\rangle + \frac{1}{\sqrt{6}}|1 - 1\ 1\ 1\rangle$$

Probability amplitudes - CG coefficients

Clebsch-Gordan coefficients

To deal with CG coefficients, in addition to counting function,

we have to introduce PATHS and MAPS

istockphoto

Paths

Path is defined by initial $s^2_{j_i,m_i,g_i,l_i}$ and final $s^2_{j_f,m_f,g_f,l_f}$ sequence

$$s^2_{j_i,m_i,g_i,l_i} \rightarrow s^2_{j_f,m_f,g_f,l_f}$$

$$\begin{pmatrix} A \\ A \\ C \\ B \\ B \\ A \end{pmatrix} \begin{matrix} \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \end{matrix} \begin{pmatrix} B \\ A \\ A \\ B \\ B \\ D \end{pmatrix}$$

Total number of paths is simply a product

$$\Phi_{\text{path}} = \Phi(j_i, m_i, g_i, l_i)\,\Phi(j_f, m_f, g_f, l_f)$$

Maps

Map connects initial and final sequence using addition modulo 2

$$s^2_{j_i,m_i,g_i,l_i} \oplus s^2_{map} = s^2_{j_f,m_f,g_f,l_f}$$

$0 \oplus 0 = 0 \qquad 1 \oplus 1 = 0$

$0 \oplus 1 = 1 \qquad 1 \oplus 0 = 1$

$A \oplus A = 00 \oplus 00 = 00 = A$

$A \oplus B = 00 \oplus 11 = 11 = B$ ...

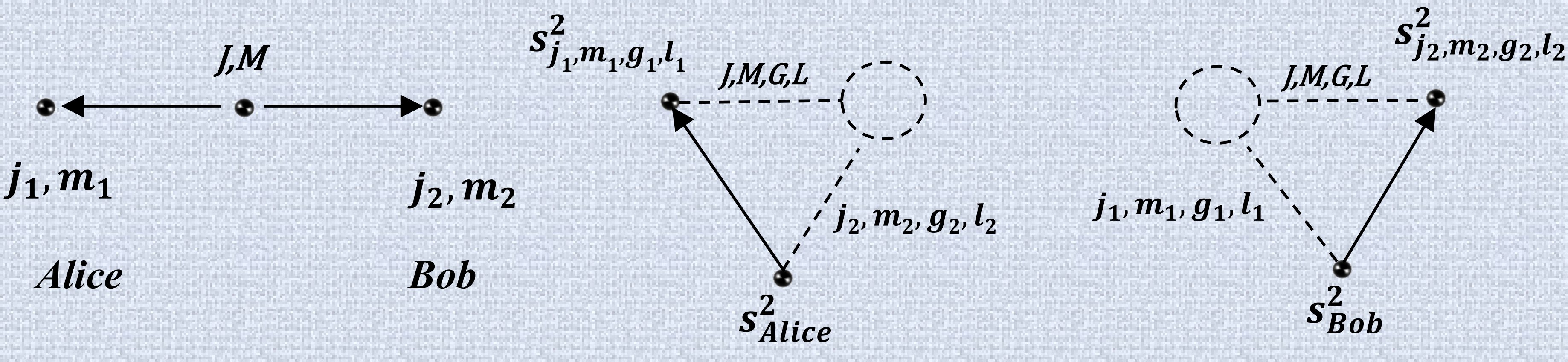
$$\begin{pmatrix} A \\ A \\ C \\ B \\ B \\ A \end{pmatrix} \oplus \begin{pmatrix} B \\ A \\ C \\ A \\ A \\ D \end{pmatrix} = \begin{pmatrix} B \\ A \\ A \\ B \\ B \\ D \end{pmatrix}$$

$$s^2_{\frac{1}{2},+\frac{1}{2},\frac{5}{2},+\frac{1}{2}} \oplus s^2_{1,0,2,1} = s^2_{\frac{1}{2},-\frac{1}{2},\frac{5}{2},-\frac{1}{2}}$$

Clebsch-Gordan coefficients

CGCs can be interpreted as measurements by two different observers

Particle *(J,M,G,L)* decays into j_1 measured by Alice and j_2 measured by Bob



Alice measures j_1,m_1 and knowing *J,M* infers all the possibilities Bob could see $\sum \Phi_B(k_{0B})$

Bob measures j_2,m_2 and knowing *J,M* infers all the possibilities Alice could see $\sum \Phi_A(k_{0A})$

Re-localization: Map Alice into Bob (they compare the notes)

interference!

$$\Phi_{path} = \sum_{k_{0A},k_{0B}} e^{i(k_{0A}-k_{0B})\pi}\, \Phi_A(k_{0A})\Phi_B(k_{0B})$$

$\boldsymbol{k_0 = \widetilde{010}}$ is a non-local element – cannot be determined from J,M, j_1,m_1, j_2,m_2 ...

Clebsch-Gordan coefficients

Normalize to get the CGCs

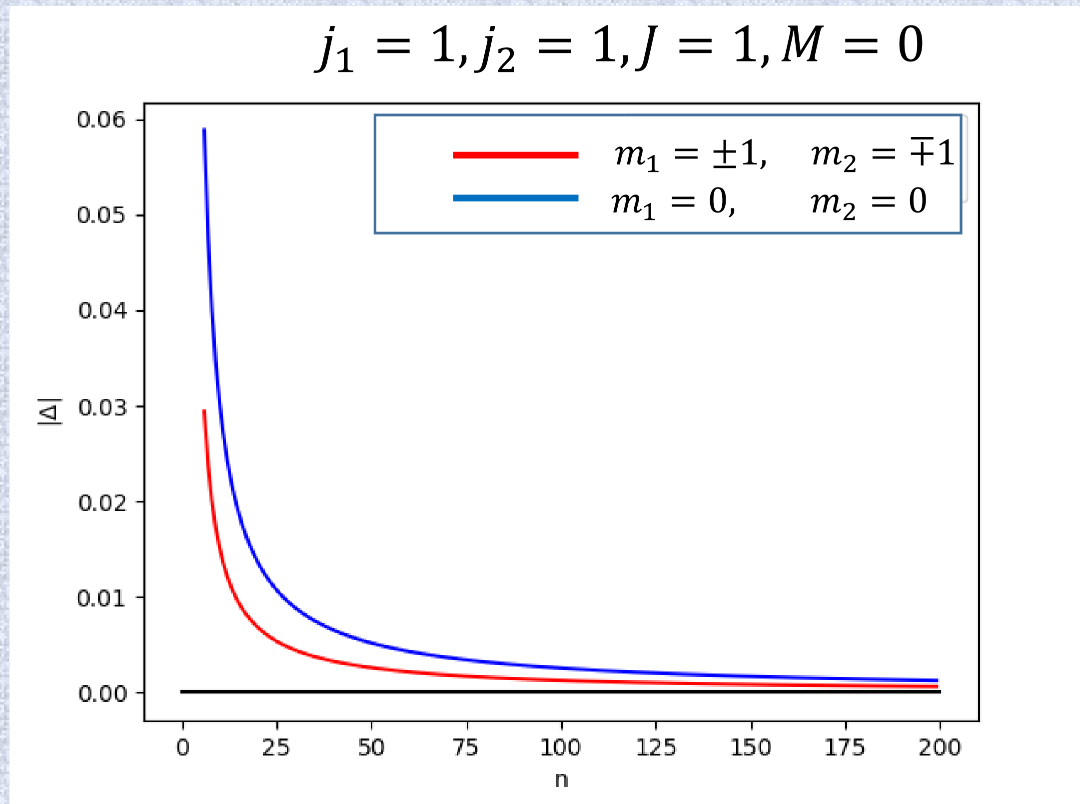
$$|\mathrm{CGC}|^2 = \frac{\Phi_{path}}{\sum\limits_{m_1,m_2} \Phi_{path}} = |\langle j_1 m_1 j_2 m_2 |\, j_1 j_2 J M \rangle|^2$$

$$< j_1 j_2 J M | j_1 j_2 m_1 m_2 >= \frac{\sum_{\widetilde{010}} \frac{e^{-i\widetilde{010}\pi} \sqrt{(\widetilde{101}+\widetilde{100})!(\widetilde{010}+\widetilde{011})!(\widetilde{110}+\widetilde{010})!(\widetilde{001}+\widetilde{101})!}}{\widetilde{101}!\widetilde{100}!\widetilde{010}!\widetilde{011}!\widetilde{110}!\widetilde{001}!}}{\sum_{m_1,m_2} \sum_{\widetilde{010}} \frac{e^{-i\widetilde{010}\pi} \sqrt{(\widetilde{101}+\widetilde{100})!(\widetilde{010}+\widetilde{011})!(\widetilde{110}+\widetilde{010})!(\widetilde{001}+\widetilde{101})!}}{\widetilde{101}!\widetilde{100}!\widetilde{010}!\widetilde{011}!\widetilde{110}!\widetilde{001}!}}$$

$$< j_1 j_2 J M | j_1 j_2 m_1 m_2 >= \frac{\sum_{k_0} \frac{e^{-ik_0\pi} \sqrt{(j_1+m_1)!(j_1-m_1)!(j_2+m_2)!(j_2-m_2)!}}{(j_1+j_2-J-k_0)!(m_1-j_2+J+k_0)!(j_2+m_2-k_0)!(-m_2-j_1+J+k_0)!}}{\sum_{m_1,m_2} \sum_{k_0} \frac{e^{-ik_0\pi} \sqrt{(j_1+m_1)!(j_1-m_1)!(j_2+m_2)!(j_2-m_2)!}}{(j_1+j_2-J-k_0)!(m_1-j_2+J+k_0)!(j_2+m_2-k_0)!(-m_2-j_1+J+k_0)!}}$$

Comparison with QM

$$|\mathrm{CGC}|^2 = \frac{\Phi_{path}}{\sum \Phi_{path}} = |\langle j_1 m_1 j_2 m_2 | j_1 j_2 J M\rangle|^2$$



Difference between our formalism and QM as a function of n

$\Delta \to 0 \quad as \quad n \to \infty$

Quantum mechanics is $n \to \infty$ limit of this formalism?

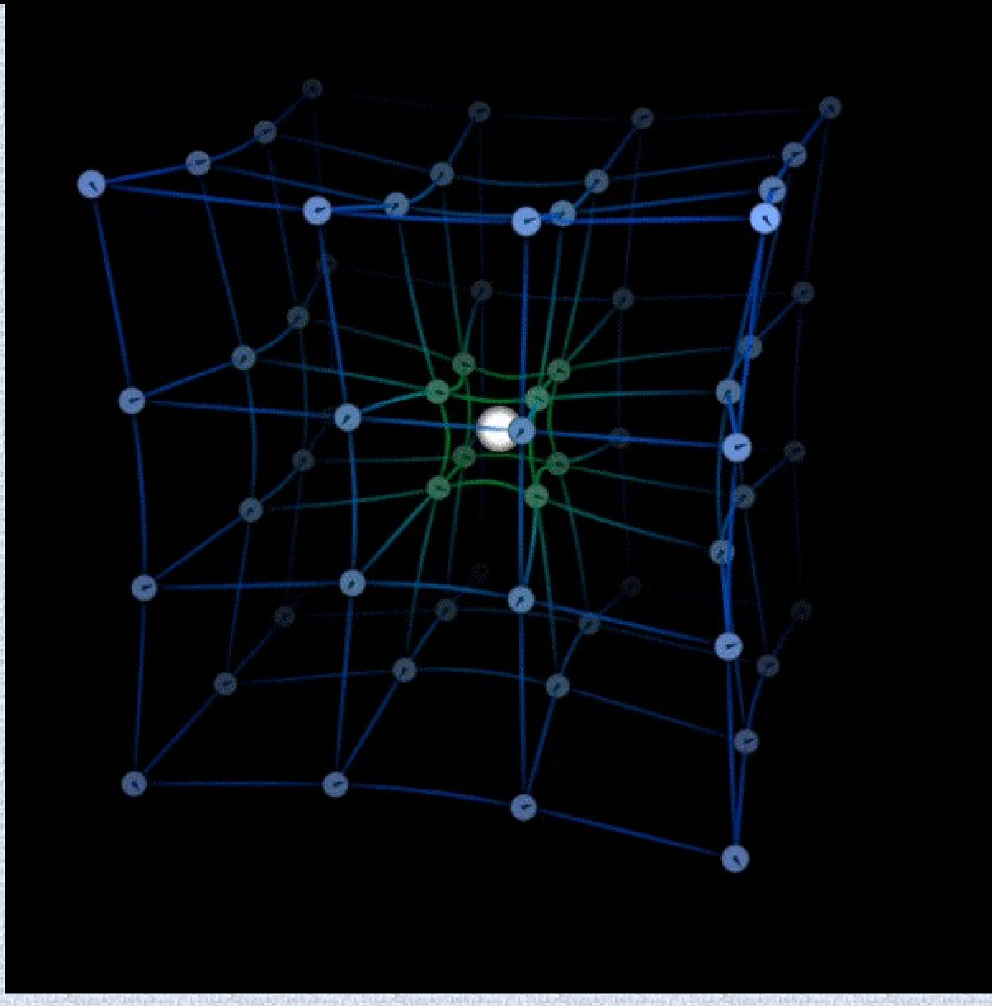
Clebsch-Gordan coefficients

$$|\mathrm{CGC}|^2 = \frac{\Phi_{path}}{\sum_{m_1, m_2} \Phi_{path}}$$

- We reduced QM probability to counting permutations (no Born rule)
- It is now clear that we have to square the wavefunction to get probability

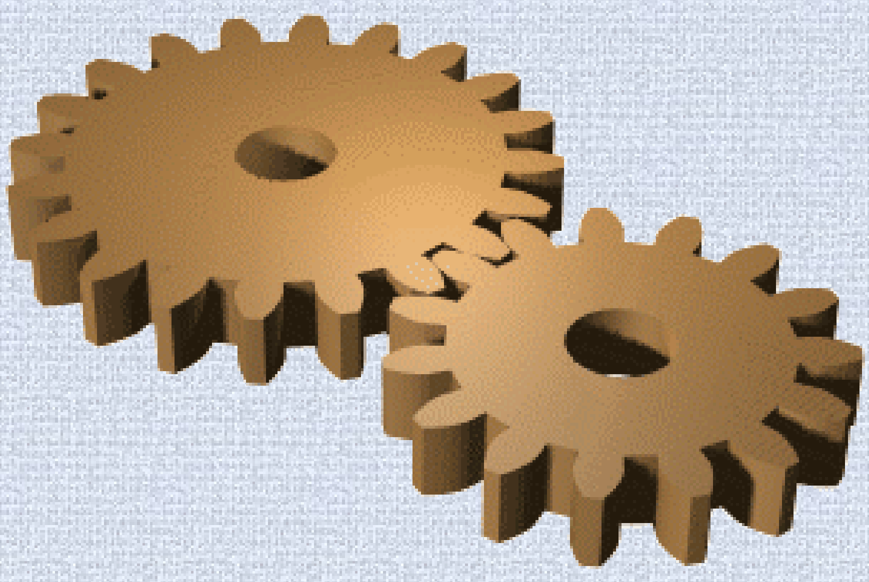
CGC contain all the QM – probabilities, interference, non-locality!

How do space and time emerge?



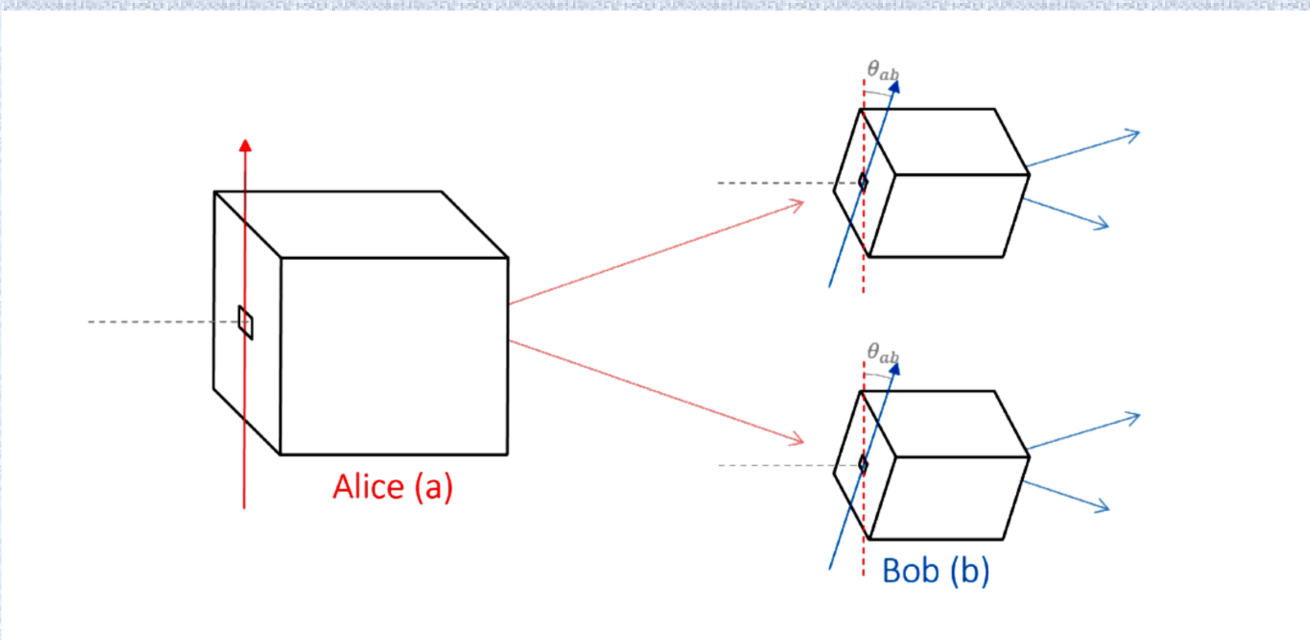
- We derived all the results so far without any reference to spacetime
- We need input from experiments to proceed

Rotations



First step toward emergent spacetime!

Two sequential Stern-Gerlach experiments



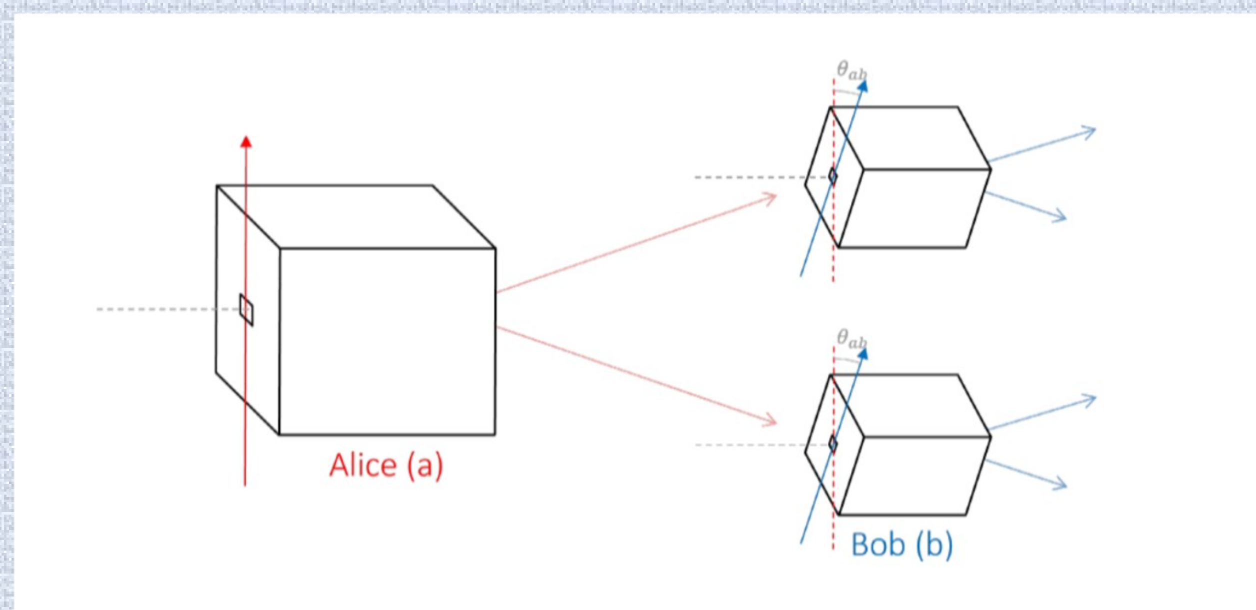
Two events

Event 1 occurs at Alice's detector which deflects j = 1/2 particle into one of two paths

Bob then rotates his detector with respect to Alice's by the angle $\boldsymbol{\theta_{ab}}$.

Event 2 occurs at Bob's detector which deflects j = 1/2 particle into one of two paths

Two sequential Stern-Gerlach experiments



- If Alice measures the spin projection m, what is the probability for Bob to measure m′, if his apparatus is rotated by an angle θ?
- For 100 years there was only one way to answer this question (QM)
- Now there are two!

QM prediction

Wigner's d-matrix formula:

Given total spin **j** and initial spin projection **m**, the probability of observing **m′**, under relative rotation of spatial frames by **θ**, is

$$\left(d^j_{m',m}(\theta)\right)^2 = \sum_{q^a}(-1)^{m'-m+q^a}\frac{(j+m)!(j-m)!}{(j+m-q^a)!q^a!(m'-m+q^a)!(j-m'-q^a)!}$$
$$\times\left(cos(\frac{\theta}{2})\right)^{2j+m-m'-2q^a}\left(sin(\frac{\theta}{2})\right)^{m'-m+2q^a}$$
$$\times\sum_{q^b}(-1)^{m'-m+q^b}\frac{(j+m')!(j-m')!}{(j+m-q^b)!q^b!(m'-m+q^b)!(j-m'-q^b)!}$$
$$\times\left(cos(\frac{\theta}{2})\right)^{2j+m-m'-2q^b}\left(sin(\frac{\theta}{2})\right)^{m'-m+2q^b}$$

Modeling Rotations

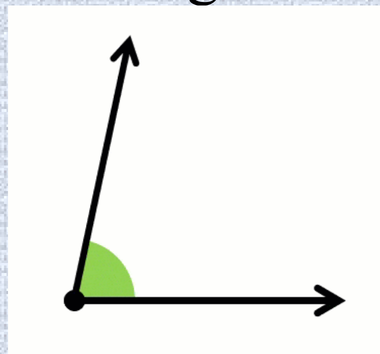
Require Alice and Bob to see the same value of *j*, but *m* can change

Thus, maps between Alice and Bob can contain only A’s and B’s

$$\begin{aligned}
A \oplus A &= B \oplus B = C \oplus C = D \oplus D = A \\
A \oplus B &= B \oplus A = C \oplus D = D \oplus C = B \\
A \oplus C &= C \oplus A = B \oplus D = D \oplus B = C \\
A \oplus D &= D \oplus A = B \oplus C = C \oplus B = D
\end{aligned}$$

$$\theta_{ab} \equiv \frac{\widetilde{B}_{map}}{n}\pi$$

Angle



Example n=2

Alice observes $\boldsymbol{m_a} = +\frac{1}{2}$, $\boldsymbol{l_a} = +\frac{1}{2}$ and we rotate by $\theta_{ab} = \pi/2$

We then apply all possible maps associated with $n = 2$ and $\theta_{ab} = \pi/2$

$$\left\{\begin{pmatrix} C \\ A \end{pmatrix}_{a1}\right\} \oplus \left\{\begin{pmatrix} A \\ B \end{pmatrix}_{map}, \begin{pmatrix} B \\ A \end{pmatrix}_{map}\right\} = \left\{\begin{pmatrix} C \\ B \end{pmatrix}_{b2}, \begin{pmatrix} D \\ A \end{pmatrix}_{b2}\right\}$$

$$\boldsymbol{m_a} = +\frac{1}{2}\ , \qquad \boldsymbol{\theta_{ab} \equiv \frac{\widetilde{B}_{map}}{n}\pi} \qquad \boldsymbol{m_b} = +\frac{1}{2} \quad \boldsymbol{m_b} = -\frac{1}{2}$$

Bob measures $\boldsymbol{m_b} = \pm\frac{1}{2}$ with equal probability (50% each)

If we rotate by $\theta_{ab} = \pi$ *i.e.* $\begin{pmatrix} B \\ B \end{pmatrix}$, then $\boldsymbol{m_b} = -\frac{1}{2}$ with 100% probability

Matches QM predictions

Getting a general expression

Experiment consists of an event at Alice's and Bob's detectors

Ontic states are ordered pairs of base-4 sequences:
(or 4-point correlations between base-2 sequences)

Alice Bob

$$\begin{pmatrix} A & C \\ A & D \\ C & D \\ B & A \\ A & B \\ D & B \end{pmatrix}$$

They are base-16 sequences comprised of the following symbols:
{AA,AB,AC,AD,BA,BB,BC,BD,CA,CB,CC,CD,DA,DB,DC,DD}

Since rotation maps can contain only the symbols A and B

Basis for ontic states of this experiment is:

{AA,AB,BA,BB,CC,CD,DC,DD}

There is 8 of them

Converting quantum numbers into base-8 counts

Since Alice is on the left and Bob is on the right

Alice’s and Bob’s counts in terms of base-8 counts
{AA,AB,BA,BB,CC,CD,DC,DD}

are

Alice Bob

$$\begin{pmatrix} A & C \\ A & D \\ C & D \\ B & A \\ A & B \\ D & B \end{pmatrix}$$

$$\widetilde{C_a} = \widetilde{CC} + \widetilde{CD} \qquad \widetilde{D_a} = \widetilde{DC} + \widetilde{DD}$$

$$\widetilde{C_b} = \widetilde{CC} + \widetilde{DC} \qquad \widetilde{D_b} = \widetilde{CD} + \widetilde{DD}$$

...

→ Quantum numbers

$$j_a = j_b = \frac{\widetilde{C_a}+\widetilde{D_a}}{2} = \frac{\widetilde{C_b}+\widetilde{D_b}}{2} = \frac{\widetilde{CC}+\widetilde{CD}+\widetilde{DC}+\widetilde{DD}}{2}$$

$$m_a = \frac{\widetilde{C_a}-\widetilde{D_a}}{2} = \frac{\widetilde{CC}+\widetilde{CD}-\widetilde{DC}-\widetilde{DD}}{2} \qquad m_b = \frac{\widetilde{C_b}-\widetilde{D_b}}{2} = \frac{\widetilde{CC}+\widetilde{DC}-\widetilde{CD}-\widetilde{DD}}{2}$$

Non-local quantum numbers

We can write 7 quantum numbers $\boldsymbol{n, j, m_a, m_b, l_a, l_b, \theta_{ab}}$ in terms of basis {AA,AB,BA,BB,CC,CD,DC,DD}

We need the 8th one. Define

$$\boldsymbol{\mu_{a,b}} = \frac{\widetilde{\boldsymbol{CD}} + \widetilde{\boldsymbol{DC}} + \widetilde{\boldsymbol{AA}} + \widetilde{\boldsymbol{BB}}}{\boldsymbol{2}}$$

- $\boldsymbol{\mu_{a,b}}$ is a non-local quantum number (property of the whole experiment)
- Not associated with Alice's and Bob's events, nor with the map
- Alice's and Bob's ensembles can disagree about their values

Probability is proportional to cardinality

For each unique combination of eight quantum numbers, ε^a and ε^b are ontic states where Alice's (Bob's) event is held fixed

$$n = 4, j = \frac{1}{2}, m_a = \frac{1}{2}, m_b = \frac{1}{2}, l_a = \frac{1}{2}, l_b = \frac{1}{2}, \theta_{ab} = \frac{\pi}{2}, \mu_{ab} = \frac{1}{2}$$

$$\varepsilon^a = \begin{pmatrix} C \\ B \\ A \\ A \end{pmatrix}_{a1} \otimes \left\{ \begin{pmatrix} C \\ A \\ A \\ B \end{pmatrix}_{b2}, \begin{pmatrix} C \\ A \\ B \\ A \end{pmatrix}_{b2} \right\}, \quad \varepsilon^b = \left\{ \begin{pmatrix} A \\ A \\ C \\ B \end{pmatrix}_{a1}, \begin{pmatrix} A \\ B \\ C \\ A \end{pmatrix}_{a1} \right\} \otimes \begin{pmatrix} B \\ A \\ C \\ A \end{pmatrix}_{b2}$$

Cardinality of ε^a and ε^b

$$|\epsilon^a| = \frac{\widetilde{A_a}!\ \widetilde{B_a}!\ \widetilde{C_a}!\ \widetilde{D_a}!}{\widetilde{AA}!\ \widetilde{AB}!\ \widetilde{BA}!\ \widetilde{BB}!\ \widetilde{CC}!\ \widetilde{CD}!\ \widetilde{DC}!\ \widetilde{DD}!} \qquad |\epsilon^b| = \frac{\widetilde{A_b}!\ \widetilde{B_b}!\ \widetilde{C_b}!\ \widetilde{D_b}!}{\widetilde{AA}!\ \widetilde{AB}!\ \widetilde{BA}!\ \widetilde{BB}!\ \widetilde{CC}!\ \widetilde{CD}!\ \widetilde{DC}!\ \widetilde{DD}!}$$

Combined cardinality $\left|\epsilon^a \otimes \epsilon^b\right| = \left|\epsilon^a\right|\left|\epsilon^b\right|$ ➡ **number of paths**

Interference

Pairs in Alice's and Bob's ensembles can have different $\boldsymbol{\mu_{ab}}$

$$n = 6, j = 1, m_a = 0, m_b = 0, l_a = 0, l_b = 1, \theta_{ab} = \frac{\pi}{2}$$

$$\begin{pmatrix} AA \\ BB \\ DC \\ CD \\ BA \\ AA \end{pmatrix}_{\mu_{a1,b2}=\frac{5}{2}}, \quad \begin{pmatrix} AB \\ BA \\ DD \\ CC \\ BA \\ AA \end{pmatrix}_{\mu_{a1,b2}=\frac{1}{2}}$$

- Pairs with an odd value of $\boldsymbol{\Delta\mu_{ab}}$ annihilate pairs with an even value

$$\sum_{\mu^a_{ab}, \mu^b_{ab}} (-1)^{\Delta\mu_{ab}} \left|\epsilon^a(\mu^a_{ab})\right| \left|\epsilon^b(\mu^b_{ab})\right|$$

Interference!

$\Delta\mu_{ab}$ related to **q** from Wigner's d-matrix

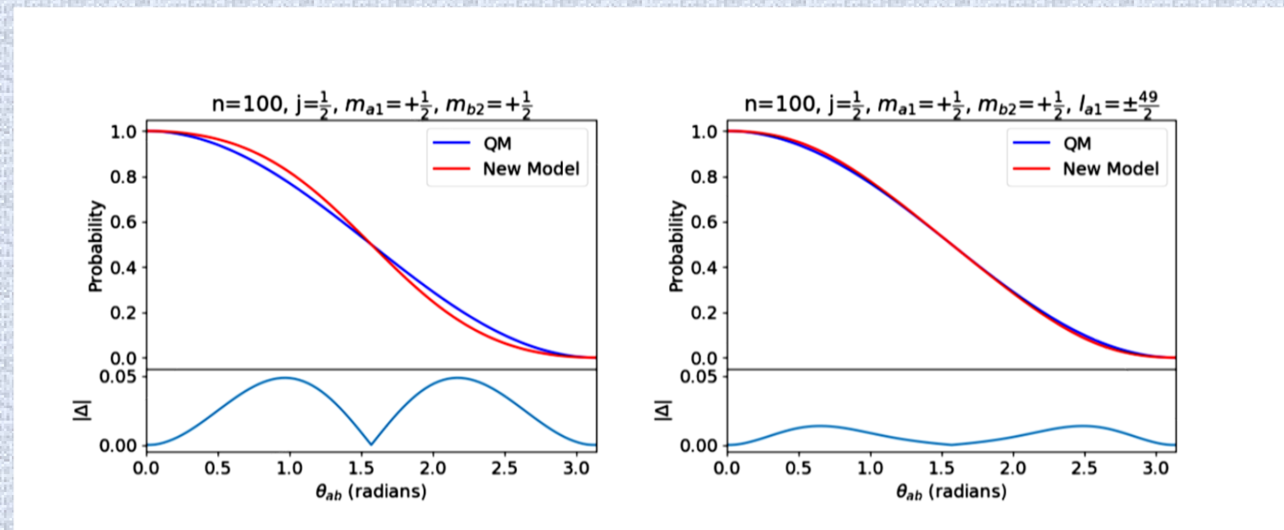
Total Probability

Υ is the total cardinality $|\epsilon^a \otimes \epsilon^b| = |\epsilon^a||\epsilon^b|$ after we account for interference and sum up over l's

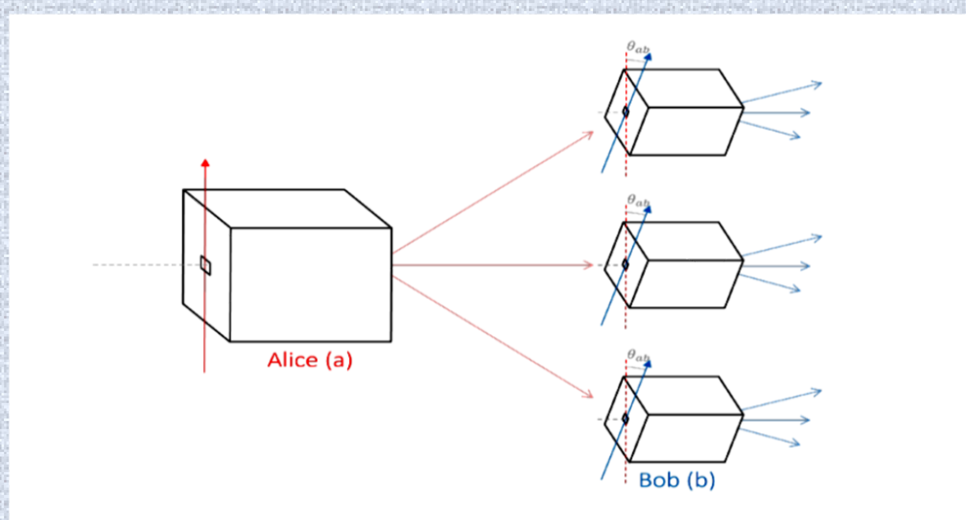
Finally, the probability is given as:

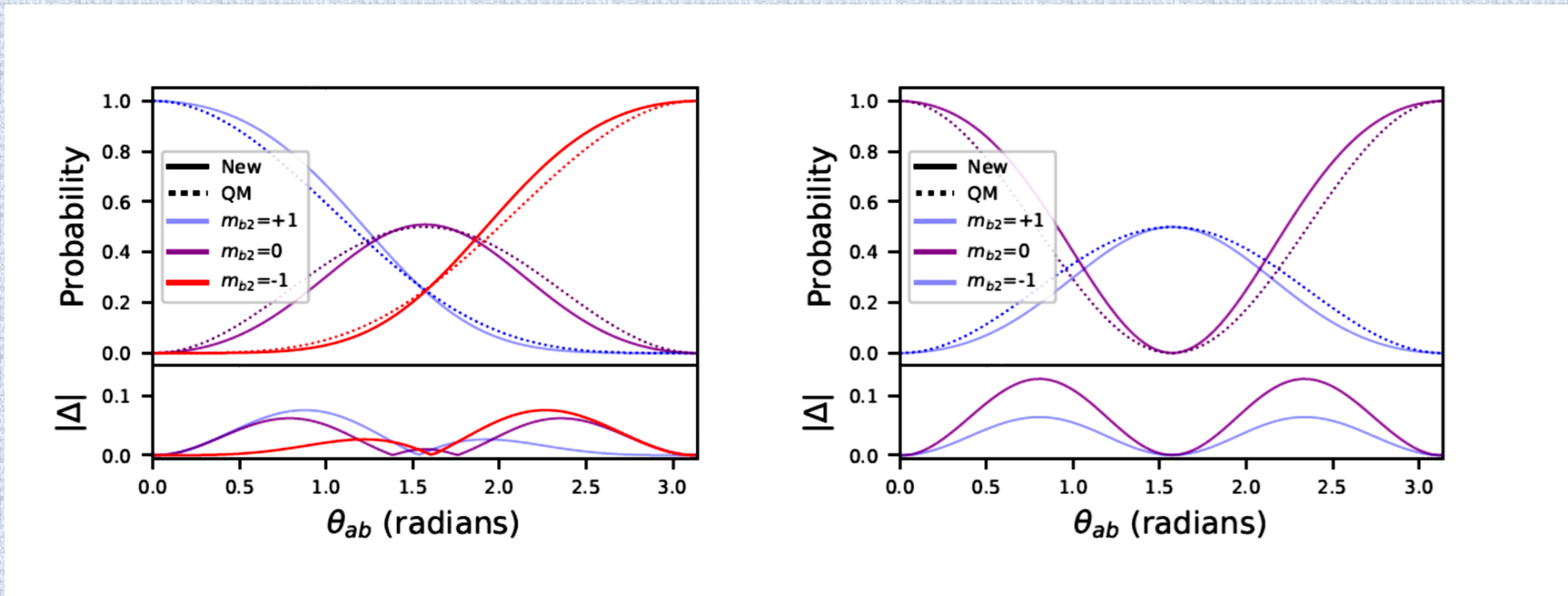
$$P\left(m_b \middle| n, j = \frac{1}{2}, m_a, \theta_{ab}\right) = \frac{\Upsilon\left(n, j = \frac{1}{2}, m_a, m_b, \theta_{ab}\right)}{\sum_{m_b} \Upsilon\left(n, j = \frac{1}{2}, m_a, m_b, \theta_{ab}\right)}$$

- Compare P with QM predictions (Wigner's d-matrix formula)



Spin 1 particles

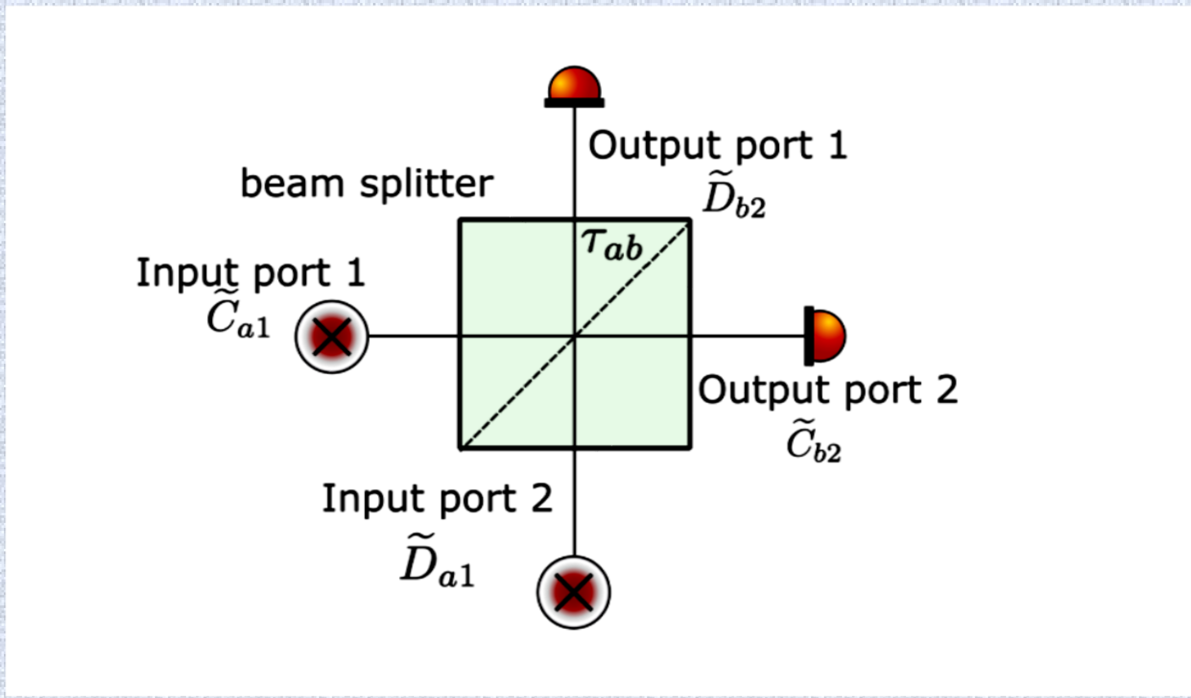




n = 100, j = 1, m_a = +1 (left), and m_a = 0 (right), while Δ is the difference

Optical systems

- Optical systems are much easier to work with than spin systems
- Work with photon number states (Fock states)



- Photon beam splitter with transmittance $\tau_{ab} = \cos^2\left(\underbrace{\frac{\widetilde{B}_{map}}{n}\frac{\pi}{2}}_{\frac{\theta}{2}}\right)$

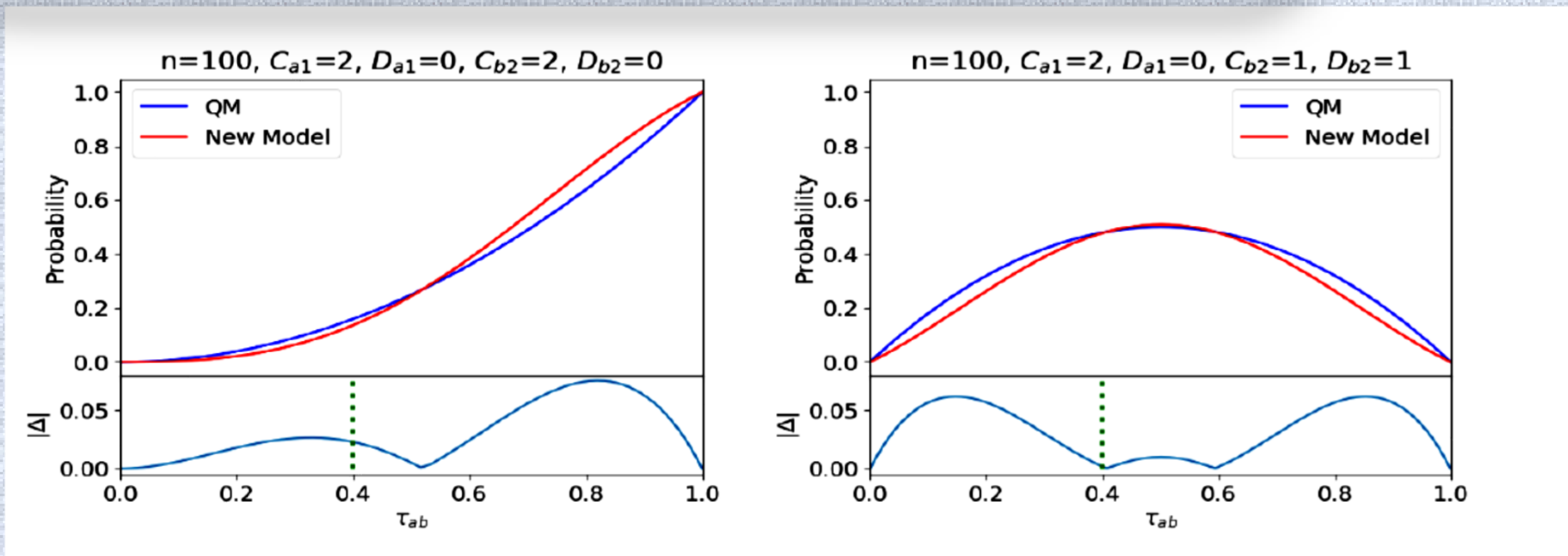
- C's and D's can represent ladder operators n_+ and n_-

Results

Impose conservation of the number of photons: $\widetilde{C_a} + \widetilde{D_a} = \widetilde{C_b} + \widetilde{D_b}$

$$\widetilde{C_a} = \widetilde{CC} + \widetilde{CD} \qquad \widetilde{D_a} = \widetilde{DC} + \widetilde{DD}$$
$$\widetilde{C_b} = \widetilde{CC} + \widetilde{DC} \qquad \widetilde{D_b} = \widetilde{C_a} + \widetilde{D_a} - \widetilde{C_b}$$

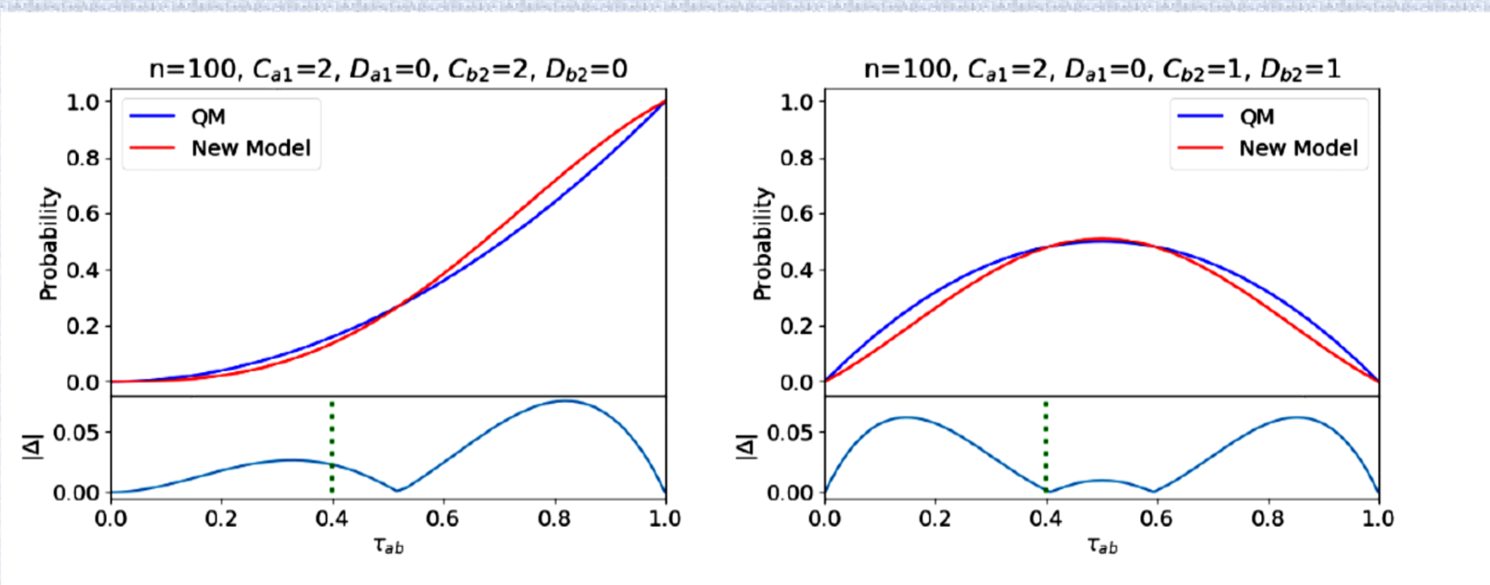
$$\boldsymbol{P}\left(\widetilde{C_b}, \widetilde{D_b} \middle| \boldsymbol{n}, \widetilde{C_a}, \widetilde{D_a}, \boldsymbol{\tau_{ab}}\right) = \frac{\boldsymbol{\Upsilon}\left(\boldsymbol{n}, \widetilde{C_a}, \widetilde{D_a}, \widetilde{C_b}, \boldsymbol{\tau_{ab}}\right)}{\sum_{\widetilde{C_b}} \boldsymbol{\Upsilon}\left(\boldsymbol{n}, \widetilde{C_a}, \widetilde{D_a}, \widetilde{C_b}, \boldsymbol{\tau_{ab}}\right)}$$



Difference goes down with n, but does not disappear (like in CGC)

Results

There is always a difference between our formalism and QM for finite n



- If rotations are involved, further difference is present
- Perhaps because spacetime is not classical like in QM
- I.e. rotations are probabilistic

Summary

We derived angular momentum rules in QM from binary sequences
Also analogs of CGC and Wigner's d-matrix formula

Along the way we learned:

- Particles are relationships between the sequences (emergent phenomena)
- Obscuring information about the sequences leads to non-determinism
- World is non-deterministic if we (humans) see particles and fields
- For a super-observer seeing sequences, world is deterministic
- Observer (reference sequence) is an integral part of the system

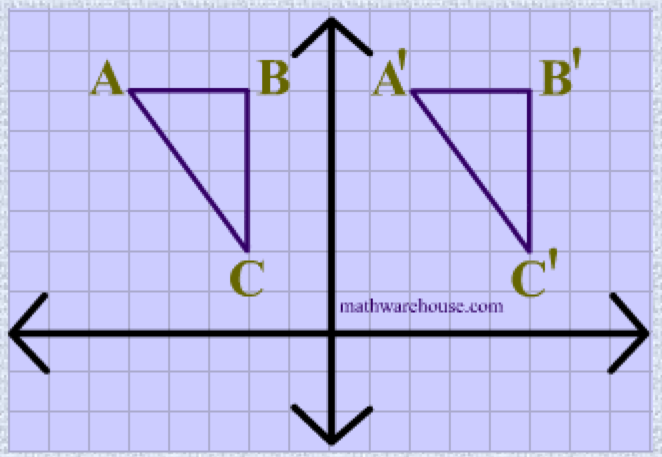
Summary

- We reduced QM probabilities to counting
- Counting unit is $\frac{\hbar}{2}$ which perhaps explains $\frac{1}{2}$ in $\Delta p \Delta x \geq \frac{\hbar}{2}$
  (also accommodates fermions)
- It is clear that we have to square the WF to get probability in QM
- To recover QM, limit $n \to \infty$ is required
  (opens the door to test this formalism)
- Precise quantum optics experiments may find deviations from QM

Next

To see how space and time arise in this formalism

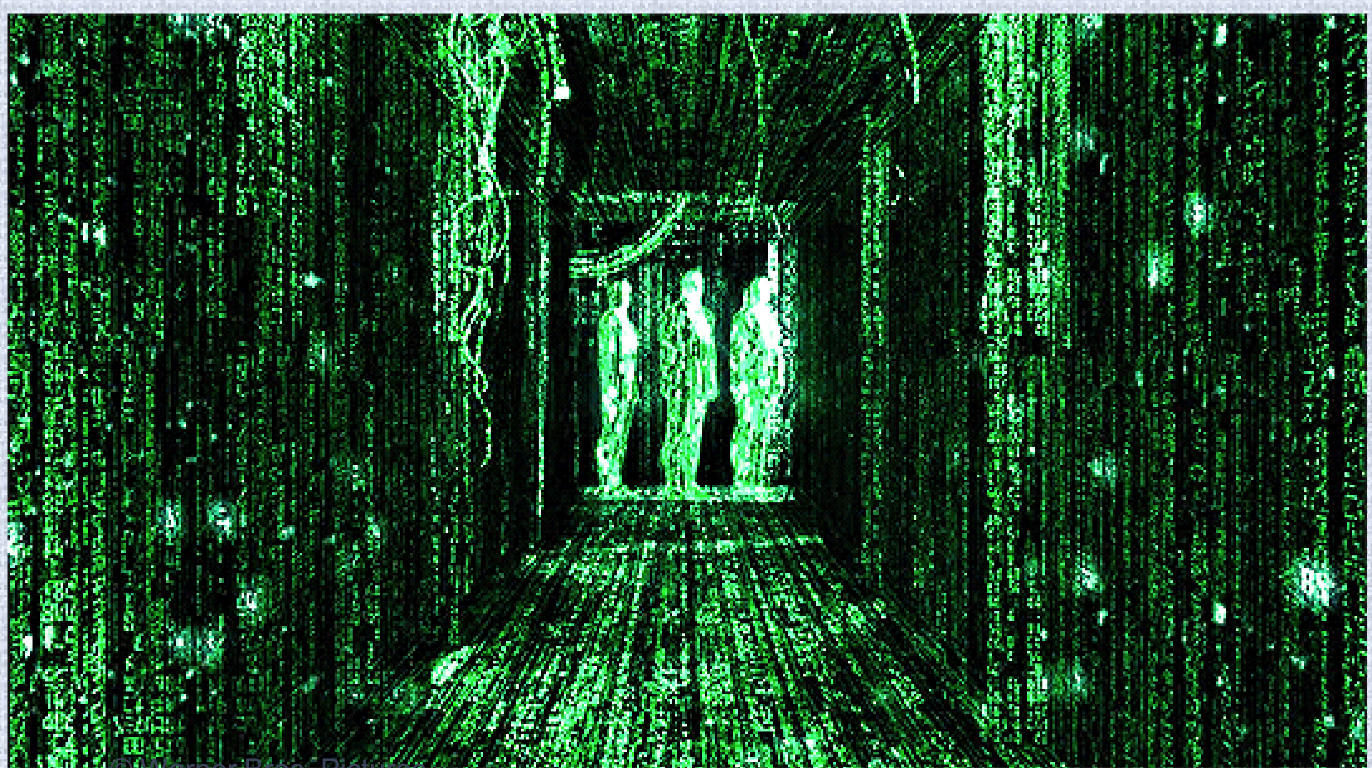
- We already found a way to describe rotations is space
- Need to add translations in space and translations in time



- Emergent spacetime!

Trillion-dollar question

Do we live in a simulation???



Possible, but not necessary...


Thank you