Tensor Gauge Field Theory and Extension of Chern-Simons Form

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Publications

- 1. *Interaction of non-Abelian tensor gauge fields Arm.J.Math.* 1 (2008) 1-17 G.S.
- 2. *Extension of the Poincaré Group and Non-Abelian Tensor Gauge Fields Int.J.Mod.Phys.A* 25 (2010) 5765-5785 G.S
- 3. *Extensions of the Poincare group. J.Math.Phys.* 52 (2011) 072303I.Antoniadis, L.Brink, G.S.

——————-

- 4. *Topological mass generation in four-dimensional gauge theory. Phys.Lett.B* 694 (2011) 65-73 G.S.
- 5. *New gauge anomalies and topological invariants in various dimensions. Eur. Phys. J. C* 72 (2012) 2140 **I. Antoniadis and G.S.**
- 6. *Extension of Chern-Simons forms and new gauge anomalies. Int.J.Mod.Phys.A* 29 (2014) 1450027 **I.Antoniadis and G.S.**
- 7. *Extension of Chern-Simons forms. J.Math.Phys.* 55 (2014) 062304 S.Konitopoulos and G.S.
- 8. *Asymptotic freedom of non-Abelian tensor gauge fields. Phys.Lett.B* 732 (2014) 150-155 G.S.
- 9. *Generalisation of the Yang-Mills Theory. Int.J.Mod.Phys.A* 31 (2016) 1630003 G.S. International Conference on 60 Years of Yang-Mills Gauge Field Theories

^{——————-} 10. *Lars Brink Colleague, Friend and Collaborator G.S.*

Extension of the Poincaré Group

Representations and Killing Metric

Non-Abelian Tensor Gauge Fields

Transformation of Tensor Gauge Fields

The Lagrangian

Interactions and Asymptotic Freedom

Callan-Simanzik beta function

Topological Mass Generation

Topological invariants in various dimensions

Transgression and Secondary Forms

Extension of Chern-Simons Forms - CSAS

Extension of the Poincaré Algebra Extension of the Poincaré Algebra Fxtension of the Poincaré Algebra

$$
[P^{\mu}, P^{\nu}] = 0,
$$

\n
$$
[M^{\mu\nu}, P^{\lambda}] = i(\eta^{\lambda\nu} P^{\mu} - \eta^{\lambda\mu} P^{\nu}),
$$

\n
$$
[M^{\mu\nu}, M^{\lambda\rho}] = i(\eta^{\mu\rho} M^{\nu\lambda} - \eta^{\mu\lambda} M^{\nu\rho} + \eta^{\nu\lambda} M^{\mu\rho} - \eta^{\nu\rho} M^{\mu\lambda}),
$$

\n
$$
[P^{\mu}, L^{\lambda_1...\lambda_s}_{a}] = 0,
$$

\n
$$
[M^{\mu\nu}, L^{\lambda_1...\lambda_s}_{a}] = i(\eta^{\lambda_1\nu} L^{\mu\lambda_2...\lambda_s}_{a} - \eta^{\lambda_1\mu} L^{\nu\lambda_2...\lambda_s}_{a} + ... + \eta^{\lambda_s\nu} L^{\lambda_1...\lambda_{s-1}\mu}_{a} - \eta^{\lambda_s\mu} L^{\lambda_1...\lambda_{s-1}\nu}_{a}),
$$

\n
$$
[L^{\lambda_1...\lambda_i}_{a}, L^{\lambda_{i+1}...\lambda_s}_{b}] = i f_{abc} L^{\lambda_1...\lambda_s}_{c} \quad (\mu, \nu, \rho, \lambda = 0, 1, 2, 3; \quad s = 0, 1, 2, ...),
$$

$$
L_a^{\lambda_1\cdots\lambda_s}=e^{\lambda_1}\cdots e^{\lambda_s}\otimes L_a\,,\quad s=0,1,2,\ldots\,.
$$

These generators carry space–time and internal indices and transform under the operations of both groups. The algebra of these generators \rm^6

$$
\left[L_a^{\lambda_1\cdots\lambda_i},L_b^{\lambda_{i+1}\cdots\lambda_s}\right] = if_{abc}L_c^{\lambda_1\cdots\lambda_s}\,,\quad s = 0,1,2,\ldots
$$

Extension of the Poincaré Algebra generators and states. Extension of the Poincaré Algebra

it is a "gauge invariant" extension of the Poincaré algebra in a sense that if one defines a "gauge" transformation of its generators as it is a "gauge invariant" extension of the Poincaré algebra in a sens It is a gauge invariant extension of the 1 onical algebra in a sent
defines a "gauge" transformation of its generators as ige invariant" extension
rauge" transformation o defines a "gauge" transformation of its generators as it is a "gauge invariant" extension of the Poincaré algebra in a sense th mutators (13) and (19) are essentially different in a super-order of the algebras, in super-order of the algebra $\frac{1}{2}$ ∞° −−−
| a sense t l essenti
∂ev t if one ∂e^µ \sim σ ...

$$
L_a^{\lambda_1 \cdots \lambda_s} \to L_a^{\lambda_1 \cdots \lambda_s} + \sum_{1} P^{\lambda_1} L_a^{\lambda_2 \cdots \lambda_s} + \sum_{2} P^{\lambda_1} P^{\lambda_2} L_a^{\lambda_3 \cdots \lambda_s} + \cdots + P^{\lambda_1} \cdots P^{\lambda_s} L_a,
$$

$$
M^{\mu\nu} \to M^{\mu\nu}, \quad P^{\lambda} \to P^{\lambda},
$$

The algebra $L_G(\mathcal{P})$ has a simple representation of the following form $Thes$ $e^{i\theta}$ The algebra $L_G(\mathcal{P})$ has a simple representation of the following form

$$
P^{\mu} = k^{\mu},
$$

\n
$$
M^{\mu\nu} = i \left(k^{\mu} \frac{\partial}{\partial k_{\nu}} - k^{\nu} \frac{\partial}{\partial k_{\mu}} \right) + i \left(e^{\mu} \frac{\partial}{\partial e_{\nu}} - e^{\nu} \frac{\partial}{\partial e_{\mu}} \right),
$$

\n
$$
L^{\lambda_1 \cdots \lambda_s}_{a} = e^{\lambda_1} \cdots e^{\lambda_s} \otimes L_a,
$$

\n
$$
k^2 = 0, \quad k^{\mu} e_{\mu} = 0, \quad e^2 = -1.
$$

the matrix representations of this algebra are transversal the matrix representations of this algebra are transversal the matrix representations of this algebra are transver the matrix representations of this algebra are transversal presentations or this algebra are transver

$$
P_{\lambda_1}L_a^{\lambda_1\cdots\lambda_s}=0\,.
$$

Poincar´

where $\bar{\eta}^{\lambda_1 \lambda_2}$ is the projector into the two-dimensional plane transversal to the momentum $|k^\mu|$ $\bar{\eta}^{\lambda_1 \lambda_2} = \frac{k^{\lambda_1} \bar{k}^{\lambda_2} + \bar{k}^{\lambda_1} k^{\lambda_2}}{\sqrt{7}}$ $\frac{\tau \kappa}{k \bar{k}}$ - $\eta^{\lambda_1 \lambda_2}$, $k_{\lambda_1} \bar{\eta}^{\lambda_1 \lambda_2} = k_{\lambda_2} \bar{\eta}^{\lambda_1 \lambda_2} = 0$,

Non-Abelian Tensor Gauge Fields 2 Tensor Gauge Fields and Extended Poincar´e Algebra \mathbf{a} polynomial in momentum. The particular gauge in in a particular gauge. The provided metric (3.17) is written in a particular gauge. The particular gauge in a particular gauge. The particular gauge in a particular gauge. Th $\mathcal{L}(\mathcal{$

The gauge fields are defined as rank- $(s + 1)$ tensors σ are defined as rank-(s σ 1) tensors σ 1) tensors σ $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ r ne gauge i The rewe folds are defined as replied in the dependence can be written in different gauges. The spectrum of the propagating modes does not

$$
A^a_{\mu\lambda_1...\lambda_s}(x), \qquad s = 0, 1, 2, \dots
$$

and are totally symmetric with respect to the indices $\lambda_1...\lambda_s$. A priory the tensor fields have no symmetries with respect to the first index μ . The index a numerates the generators have no symmetries with respect to the first index μ . The index a numerates the generators L_a of the Lie algebra L_a of a compact Lie group G with totally antisymmetric structure constants f_{abc} . $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ is also reminiscent to the Grassmann variable experimetric to the Grassmann variable $\begin{bmatrix} 1 & 1 \end{bmatrix}$ have no symmetries with respect to the first index μ . The index a numerates the generators I_{α} of the Lie elgebra I_{α} of a compact Lie group C with totally apticymmetric structure and are totally symmetric with respect to the indices $\lambda_1...\lambda_s$. A priory the tensor fields constants f . and are totany symmetric with respect to the muices $\lambda_1...\lambda_s$. A priory the tensor neius $\frac{1}{2}$ have no symmetries with respect to the mst muck μ . The muck α numerates the generators L_a of the Lie algebra L_G of a compact Lie group G with totally antisymmetric structure $\text{constants } f_{abc}.$ const. but instants to the homogeneous ∞

$$
\mathcal{A}_{\mu}(x,e) = \sum_{s=0}^{\infty} \frac{1}{s!} A^{a}_{\mu\lambda_1...\lambda_s}(x) L_a e^{\lambda_1}...e^{\lambda_s}.
$$

dices $a, \lambda_1, ..., \lambda_s$ which are labelling the gener $L_a^{\lambda_1...\lambda_s} = L_a e^{\lambda_1}...e^{\lambda_s}$ of extended current algebra L_g associated with the Lie algebra The gauge transformation of the field $\mathcal{A}_{\mu}(x, e)$ is defined as The gauge field A^a , carries indices $a, \lambda_1, ..., \lambda_s$ which are labelling the generators The gauge field $A^a_{\mu\lambda_1...\lambda_s}$ carries indices $a, \lambda_1, ..., \lambda_s$ which are labelling the generators The gauge transformation of the field $\mathcal{A}_{\mu}(x, e)$ is defined as The gauge held $A_{\mu\lambda_1}$
 $L^{\lambda_1...\lambda_s} = L_e e^{\lambda_1} e^{\lambda_s}$

$$
\mathcal{A}'_{\mu}(x,e) = U(\xi)\mathcal{A}_{\mu}(x,e)U^{-1}(\xi) - \frac{\imath}{g}\partial_{\mu}U(\xi) U^{-1}(\xi),
$$

It is useful to have an explicit which the transform **Transformation of Tensor Gauge Fields**

 $\frac{1}{2}$ $\mathcal{L}_{\mathcal{L}}$ It is useful to have an explicit expression for the transformation law of the field components It is useful to have an explicit expression for the transformation law of the field components

$$
\delta A^a_\mu = (\delta^{ab}\partial_\mu + gf^{acb}A^c_\mu)\xi^b,
$$

\n
$$
\delta A^a_{\mu\nu} = (\delta^{ab}\partial_\mu + gf^{acb}A^c_\mu)\xi^b_\nu + gf^{acb}A^c_{\mu\nu}\xi^b,
$$

\n
$$
\delta A^a_{\mu\nu\lambda} = (\delta^{ab}\partial_\mu + gf^{acb}A^c_\mu)\xi^b_{\nu\lambda} + gf^{acb}(A^c_{\mu\nu}\xi^b_\lambda + A^c_{\mu\lambda}\xi^b_\nu + A^c_{\mu\nu\lambda}\xi^b),
$$

variant derivatives $\nabla^{ab} = (\partial_{\mu} - ia A_{\mu}(x, e))^{ab}$ covariant derivatives $\nabla_{\mu}^{ab} = (\partial_{\mu} - ig \mathcal{A}_{\mu}(x, e))^{ab}$

$$
[\nabla_\mu,\nabla_\nu]^{ab}=gf^{acb}\mathcal{G}^c_{\mu\nu} ,
$$

field strengths tensors take the following form Ga = ∂µA^a ^ν [−] [∂]νA^a ^µ + gf abc A^b ^µ A^c ^ν, (4.6) field strengths tensors take the following form

$$
G_{\mu\nu}^{a} = \partial_{\mu}A_{\nu}^{a} - \partial_{\nu}A_{\mu}^{a} + gf^{abc} A_{\mu}^{b} A_{\nu}^{c},
$$

\n
$$
G_{\mu\nu,\lambda}^{a} = \partial_{\mu}A_{\nu\lambda}^{a} - \partial_{\nu}A_{\mu\lambda}^{a} + gf^{abc} (A_{\mu}^{b} A_{\nu\lambda}^{c} + A_{\mu\lambda}^{b} A_{\nu}^{c}),
$$

\n
$$
G_{\mu\nu,\lambda\rho}^{a} = \partial_{\mu}A_{\nu\lambda\rho}^{a} - \partial_{\nu}A_{\mu\lambda\rho}^{a} + gf^{abc} (A_{\mu}^{b} A_{\nu\lambda\rho}^{c} + A_{\mu\lambda}^{b} A_{\nu\rho}^{c} + A_{\mu\rho}^{b} A_{\nu\lambda}^{c} + A_{\mu\lambda\rho}^{b} A_{\nu}^{c})
$$

\n
$$
\dots
$$

Lagiangiant di foncer daage incrac and
a Lagrapaion of Tanger Caugo Eiglde **Lagrangian of Tensor Gauge Fields**

$$
\mathcal{L}(x) = \langle \mathcal{L}(x, e) \rangle = -\frac{1}{4} \langle \mathcal{G}^{a}_{\mu\nu}(x, e) \mathcal{G}^{a\mu\nu}(x, e) \rangle,
$$

$$
\mathcal{L}(x, e) = \sum_{s=0}^{\infty} \frac{1}{s!} \mathcal{L}_{\lambda_1...\lambda_s}(x) e^{\lambda_1} ... e^{\lambda_s}.
$$

$$
\mathcal{L}(x) = \langle \mathcal{L}(x, e) \rangle = \sum_{s=0}^{\infty} \frac{1}{s!} \mathcal{L}_{\lambda_1 \dots \lambda_s}(x) \langle e^{\lambda_1} \dots e^{\lambda_s} \rangle
$$
 using Killing metric

The gauge invariant density the following form in the following form $\frac{1}{3}$ and the density for the lower-rank tensor fields is α α β β β β β ∞ $\frac{1}{2}$

$$
\mathcal{L}_2=-\frac{1}{4}G^a_{\mu\nu,\lambda}G^a_{\mu\nu,\lambda}-\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu,\lambda\lambda}.
$$

ion of Toppar C *Lagrangian of Tensor Gauge Fields*

$$
\mathcal{L}^{'}(x)=\langle\mathcal{L}^{'}(x,e)\rangle=\frac{1}{4}\langle\mathcal{G}^{a}_{\mu\rho_{1}}(x,e)e^{\rho_{1}}\left.\mathcal{G}^{a\mu}_{\qquad\rho_{2}}(x,e)e^{\rho_{2}}\right\rangle'.
$$

$$
\mathcal{L}'(x) = \langle \mathcal{L}'(x, e) \rangle = \sum_{s=0}^{\infty} \frac{1}{s!} (\mathcal{L}'_{\rho_1 \rho_2})_{\lambda_1 \dots \lambda_s}(x) \langle e^{\rho_1} e^{\rho_2} e^{\lambda_1} ... e^{\lambda_s} \rangle'.
$$

$$
\mathcal{L}_2^{'}=\frac{1}{4}G^a_{\mu\nu,\lambda}G^a_{\mu\lambda,\nu}+\frac{1}{4}G^a_{\mu\nu,\nu}G^a_{\mu\lambda,\lambda}+\frac{1}{2}G^a_{\mu\nu}G^a_{\mu\lambda,\nu\lambda}.
$$

$\frac{1}{2}$ $\overline{\mathbf{r}}$ d_a _v σ_τ <u>)</u> *n*, $\frac{1}{2}$ νıν
Θ $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ \overline{a} of Tensor Gauge Fie *Lagrangian of Tensor Gauge Fields* 4 4 2 Lagiangian of it is gauge intius

$$
L=\mathcal{L}+\mathcal{L}^{'}=-\frac{1}{4}\langle\mathcal{G}^a_{\mu\nu}(x,e)\mathcal{G}^{a\mu\nu}(x,e)\rangle+\frac{1}{4}\langle\mathcal{G}^a_{\mu\rho_1}(x,e)e^{\rho_1}|\mathcal{G}^{a\mu}|_{\rho_2}(x,e)e^{\rho_2}\rangle^{'}.
$$

The Lagrangian for the lower-rank tensor gauge fields has the following form: σ σ Lagrangian for the lower-rank tensor gauge fields has the following form:

$$
\begin{array}{lcl} \mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}^{'}_2 + ... = & - \; \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} \\ & - \; \frac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\nu,\lambda} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu,\lambda\lambda} \\ & + \; \; \frac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\lambda,\nu} + \frac{1}{4} G^a_{\mu\nu,\nu} G^a_{\mu\lambda,\lambda} + \frac{1}{2} G^a_{\mu\nu} G^a_{\mu\lambda,\nu\lambda} + ... \end{array}
$$

$$
\mathcal{L}_{3} + \mathcal{L}'_{3} = -\frac{1}{4} G^{a}_{\mu\nu,\lambda\rho} G^{a}_{\mu\nu,\lambda\rho} - \frac{1}{8} G^{a}_{\mu\nu,\lambda\lambda} G^{a}_{\mu\nu,\rho\rho} - \frac{1}{2} G^{a}_{\mu\nu,\lambda} G^{a}_{\mu\nu,\lambda\rho\rho} - \frac{1}{8} G^{a}_{\mu\nu} G^{a}_{\mu\nu,\lambda\rho\rho} + \frac{1}{3} G^{a}_{\mu\nu,\lambda\rho} G^{a}_{\mu\lambda,\nu\rho} + \frac{1}{3} G^{a}_{\mu\nu,\nu\lambda} G^{a}_{\mu\rho,\rho\lambda} + \frac{1}{3} G^{a}_{\mu\nu,\nu\lambda} G^{a}_{\mu\lambda,\rho\rho} + \frac{1}{3} G^{a}_{\mu\nu,\nu\lambda} G^{a}_{\mu\lambda,\rho\rho} + \frac{1}{3} G^{a}_{\mu\nu,\nu} G^{a}_{\mu\lambda,\lambda\rho\rho} + \frac{1}{3} G^{a}_{\mu\nu} G^{a}_{\mu\lambda,\nu\rho\rho}
$$
\n(4.18)

Interaction of Tensor Gauge Fields

Figure 1: The interaction vertex for the vector gauge boson V and two tensor gauge bosons T - the VTT vertex - $\mathcal{V}^{abc}_{\alpha\dot{\alpha}\beta\gamma\dot{\gamma}}(k,p,q)$ in non-Abelian tensor gauge field theory [11]. Vector gauge bosons are conventionally drawn as thin wave lines, tensor gauge bosons are thick wave lines. The Lorentz indices $\alpha \dot{\alpha}$ and momentum k belong to the first tensor gauge boson, the $\gamma\gamma$ and momentum q belong to the second tensor gauge boson, and Lorentz index β and momentum p belong to the vector gauge boson.

Figure 2: The quartic vertex with two vector gauge bosons and two tensor gauge bosons the VVTT vertex - $V_{\alpha\beta\gamma\gamma\delta\delta}^{abcd}(k, p, q, r)$ in non-Abelian tensor gauge field theory [11]. Vector wave lines. The Lorentz indices $\gamma\dot{\gamma}$ and momentum q belong to the first tensor gauge boson, $\delta\acute{\delta}$ and momentum r belong to the second tensor gauge boson, the index α and n second vector gauge $\overline{\mathcal{U}}$ p belong to the second vector gauge boson. gauge bosons are conventionally drawn as thin wave lines, tensor gauge bosons are thick momentum k belong to the first vector gauge boson and Lorentz index β and momentum

Callan-Simanzik Beta Function is, at large transfer momentum the strong coupling constant tends to zero faster compared to the standard case of the

$$
b = \frac{(12s^2 - 1)C_2(G) - 4n_f T(R)}{12\pi}, \qquad s = 1, 2, ...
$$

at $s = 1$ we are rediscovering the asymptotic freedom

beta function has the same signature as the standard gluons, which means that tensorgluns "accelerate" the asymptotic freedom (6.3) of the stron $\alpha(t)$. The contribution is increasing quadratically with the spin of the tensorgluons, that is, at large transfer momentum the strong coupling constant tends to zero faster compared to the standard case: \blacksquare $\overline{\Omega}$ ons "accelerate" the asymptotic freedom (6.3) of the strong interaction coupling constant

$$
\alpha(t) = \frac{\alpha}{1 + b\alpha \ t} \,, \tag{6.10}
$$

 $1 + \frac{1}{2} \left(\frac{1}{2} \right)$, $\frac{1}{2} \left(\frac{1}{2} \$

 $$ V ionisms in gauge transformations (31): ^Γ *(AU)* ⁼ ^Γ *(A)*. It shares therefore Chern-Pontryagin density in 4-D Yang-Mills Theory

$$
\mathcal{P}(A) = \frac{1}{4} \varepsilon^{\mu\nu\lambda\rho} \operatorname{Tr} G_{\mu\nu} G_{\lambda\rho} = \partial_{\mu} C^{\mu},
$$

which is a derivare $1-f_{th}$ definition of the strength strength tensors G_{th} is given that in the Chern-Simons topological vector current which is a derivative of the Chern–Simons topological vect **εαιρίζεται εναλ**
ΣΩ εναλ<mark>ρσ εναλρισ εναλρισ εναλρισ εν</mark> *l* is the Chern–Simons topological vector current in figure dimensions has many properties of the Chern–Pontryagin density $\frac{1}{2}$ Page 2 of 13 Eur. Phys. J. C (2012) 72:2140

$$
C^{\mu} = \varepsilon^{\mu\nu\lambda\rho} \operatorname{Tr}\left(A_{\nu}\partial_{\lambda}A_{\rho} - i\frac{2}{3}gA_{\nu}A_{\lambda}A_{\rho}\right).
$$

Topological Mass Generation invariance of a vector field does not necessarily lead to the mass \sim tations and suggested its realization in the suggested in T opological Ma Compatibility of gauge invariance and mass term in (2+1)-dimensional gauge field T anological Mass Gonoration tations and suggested in the suggested in (1+1)-dimensional gauge theory $\frac{1}{4}$ T_{α} paragument in favor of a pure gauge field theory mechanism was a dynamical mechanis ropological mass generation The argument in favor of a pure gauge field theory measurement in favor of a pure gauge field of a pure gauge field theory mechanism was a dynamical mechanism was a dynamical mechanism was a dynamical mechanism was a dynam

Deser, Jackiw and Templeton and Schonfeld Dosor Jackius and Tompleton and Schonfold t_{1} was demonstrated by Desert, $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ the set and suggested its realistical in the superiorm in α i Oeser, Jackiw and Templeton and Schonfeld does not its exci---

 \mathcal{L} and \mathcal{L} and \mathcal{L} in the \mathcal{L} who added to the 1 m $\frac{1}{2}$ who added to the YM Lagrangian a gauge invariant Chern-Simons density: who added to the YM Lagrangian a gauge invariant Chern-Simons density: $t_{\rm s}$ and suggested its realization in α in α in α in α . Come α and denotes added to the TIM Dagrangian a gauge invariant energy minions density.

$$
\mathcal{L}_{YMCS} = -\frac{1}{2} Tr G_{ij} G_{ij} + \frac{\mu}{2} \varepsilon_{ijk} Tr (A_i \partial_j A_k - ig - \frac{2}{3} A_i A_j A_k),
$$

where G_{ij} is a field strength tensor. The mass parame $-\text{for}$ \cdot m $\frac{1}{\pi}$ where G_{ij} is a neighbour density. The mass partial G_{ij} is a neighbour potential G_{ij} for the vector G_{ij} $\frac{1}{101}$ where G_{ij} is a field strength tensor. The mass parameter μ carries dimension of $[mass]^1$. The corresponding free equation of motion for the vector potential $A_i = e_i e^{ikx}$ has the form 3 a noia suivigur tensor. The mass pe extended the mass process to the mass process of the internal form rameter μ carri ${\rm form}$ is a field strength tensor. The mass parameter ${\rm i}$ Lield strengtl
∩ding free equ \mathfrak{g} r tensor. The
ation of motic tion for the vector poter itial $A_i =$ where $(k^2n + k k)$ is a finite strength of $\theta = 0$

$$
(-k^2 \eta_{ij} + k_i k_j) e_j + i\mu \varepsilon_{ijl} k_j e_l = 0
$$

In this article we suggest a similar mechanism that generates masses of the YM boson of the YM boson of the YM
In the YM boson of the YM boso

shall see, in non-Abelian tensor gauge theory [22, 23, 24] there exists a gauge invariant,

Ind shaws that the generated greitation becomes me and shows that the gauge held excreated becomes that and α shows that the and shows that the gauge here exertation secome and shows that the gauge field excitation becomes massive.

and tensor gauge bosons in (3+1)-dimensional space-time at the classical level. As we can the classical level.
As we can the classical level. As we can the classical level. As we can the classical level. As we can then th

 $\overline{}$, $\overline{}$

metric-independent density Γ in five-dimensional space-time $\frac{1}{2}$

metric-independent density Γ in five-dimensional space-time
2: The Five-dimensional space-time 2: The Five-dimensional space-time 2: The Five-dimensional space-time 2: Th

metrical metricans den space-timez: we suggest a similar mechanism that generates and tensor gauge bosons in (3+1)-dimensional space-time at the classical level. As we can at the classical level. As we we suggest a similar inechamsin that generates masses of the TIM boson we suggest a similar mechanism that generates masses of the YM boson

> which is the verified of the vector current $(0,1)$ differential space $\frac{1}{2}$ in the second density $\frac{1}{2}$ in $\frac{1}{2}$ in $\frac{1}{2}$ in the space-time 2:2: and tensor gauge bosons in $(3+1)$ -dimensional space-time at the classical level.

 $\overline{}$ = $\overline{}$ = $\overline{}$ $\overline{}$ = $\overline{}$ $\overline{}$ = $\overline{}$ $\overline{}$ = \over

New Topological Invariant in Tensor Gauge Theory New Topological Invariant in Tensor Gauge Theory and shows that the gauge field excitation becomes massive. The gauge field excitation becomes massive. The gaug
The gauge field excitation becomes massive to the gauge field excitation becomes massive. The gauge field of t Chern–Simons secondary characteristics. This construction New Topological Invariant in Tensor Gauge Theory \mathcal{A} as well as cubic and quartic terms describing nonlinear interaction of gauge fields with dimensionless coupling \mathcal{A} s, with exclusion of the terms which contain ρ ratic terms, as well as well as well as well as well as a cubic coupling of the gauge fields with dimensionless coupling of the gauge fields with dimensionless coupling of the gauge fields with dimensionless coupling of th constant *g*. In order to make all tensor gauge fields dynamical one should add all these forms as it is in the Lagrangian (4) [21–23,30,32]. *i*=1 *P P P P runs of all nonegue all nonegue all nonegue of* α $\overline{\mathcal{L}}$ with the terms which contain $\overline{\mathcal{L}}$. The term contain $\overline{\mathcal{L}}$

in non-Abelian tensor gauge theory there exists a gauge invariant, metric-independent density Γ in five-dimensional spacein non Λ bolian tensor gauge theory there exists a gauge inversiont in non-Abelian tensor gauge theory there exists a gauge invariant, metric-independent density Γ in five-dimensional space-time²: In-Abelian tensor gauge theory there exists a gauge invariant, ists a gauge invariant metric-independent density Γ (*A*) independent density Γ (*A*) independent density Γ (*A*) metric-independent density I in five-dimensional space-til ! *i*−1 *s* $\frac{1}{2}$ *µx*auge th $\overline{\mathcal{C}}$ *n*_{*i*} *s there exist a*uge invariant. $\sin \theta$: in non-Abelian tensor gauge theory there exists a gauge invariant, metric Levi-Civita epsilon tensor ε*µ*νλρσ (*µ,* ν*,...* metric-independent density Γ in five-dimensional space-time²: $time²$; **3.** Let us consider a new invariant in five-dimensional space–time *(*4 + 1*)*, which can be constructed by means of the totally antisymλρ*,*^σ *.* (8) metric macpenaent achiney **1** in nec annemne $\frac{1}{2}$ $\frac{1}{2}$

$$
\Gamma = \varepsilon_{lmnpq} Tr G_{lm} G_{np,q} = \partial_l \Sigma_l,
$$

Adding this term to the Lagrangian of non-Abelian tensor gauge fields leaves intact its

%

is the Chern-Simons topological current. Indeed, F is obviously diffeomorphism-invariant and does not involve a
In does not involve a space–time metric. It is involved a space–time metric. It is involved a space–time metri

which is the derivative of the vector current Σ_l (l,..=0,1,...,4). dimensions has many properties of the Chern-Pontryagin density P = ∂µC^µ in fourdimensions has many properties of the Chern-Pontryagin density P = ∂µC^µ in four- $1 - \varepsilon_{lmnpq} I I \cup l_m \cup_{np,q} - Q_l \cup l,$
aich is the derivative of the vector current $\Sigma_L (1 - 0.1 - 1)$ *,* (1) which is the derivative of the derivative of the vector current \mathcal{L} This invariant in five dimensions has many properties of the Chern–Pontryagin density T in five dimensions has many properties of the C the C

$$
\Sigma^l = \varepsilon^{lmnpq} \operatorname{Tr}(G_{mn} A_{pq}).
$$

which are independent on the fifth spacial coordinate $\frac{1}{2}$ gauge invariant $\frac{1}{2}$ This invariant in five dimensions has many properties o which are independent on the first spacial coordinate x4, one can get a gauge in σ dens invariant in five-dimensions has man This invenient in five dimensions hes means properties of the Chern Denturesin density This invariant in five dimensions has many properties of the Ch $\frac{1}{2}$ stream that the properties of the energy following actionly This invariant in five dimensions has many properties of the Chern–Pontryagin density *^C^µ* ⁼ ^ε*µ*νλρ Tr" 2 # This invariant in five dimensions has many p

 Γ is obviously diffeomorphism-invariant and does not involve a space-time metric. ical densities which are the non-Abelian anomalies and can has a symmetric and antisymmetric part, and only its antisymmetric part involved in (2). *Gnp,q* is the field-strength *P Price* \mathfrak{a} *I* is obviously diffeomorphism-invariant and does not involve a space–time metric. \overline{a} *js* obviously diffeomeration invariant and does not in *is* obviously diffeomorphism-invariant and

 $\overline{}$ is a construction of the state $\overline{}$ $\overline{3}$, therefore in order to get dimensionless functional in four to get dimensionless functional in four to get \overline{a} It is gauge invariant because under the gauge transionnation $\frac{1}{\sqrt{1 + \frac{1}{2}} \cdot \frac{1}{\sqrt{1 + \frac{1}{2}} \cdot \frac{1}{2}} \cdot \frac{1}{\sqrt{1 + \frac{1}{2}} \cdot \frac{1}{2}}$ $\overline{}$ It is gauge invariant because under the gauge transformation Its dimensionality is a measurement of the contract of the con 3, therefore in order to get dimensionless function \mathcal{L}_c functional in fourth \mathcal{L}_c It is gauge invariar It is gauge invariant herause under the gauge transformation die different and does not involvement and does not involve the space of the space $\frac{1}{2}$ *C*
<u>C</u>_{*n*} T_r² T_r (2) 2 *I*C IS gauge invariant because unuer the gate It is gauge invariant because under the gauge transformation It is

 $\overline{}$ invariant because under the gauge transformation $\overline{}$ is vanishes: $\overline{}$ it vanishes: $\overline{}$

A^ν ∂λ *A*^ρ − *i*

The topological character of these densities has physical

 $\overline{}$ invariant because under the gauge transformation $\overline{}$

$$
\delta_{\xi} \Gamma = -ig \varepsilon_{\mu\nu\lambda\rho\sigma} \operatorname{Tr} \bigl([G_{\mu\nu}\xi] G_{\lambda\rho,\sigma} + G_{\mu\nu} \bigl([G_{\lambda\rho,\sigma}\xi] + [G_{\lambda\rho}\xi_{\sigma}] \bigr) \bigr) = 0.
$$

Adding this term to the Lagrangian of non-Abelian tensor gauge fields leaves intact its

δ*A w* Topological In *New Topological Invariant in Tensor Gauge Theory* in Appendix A, one can see that Γ gets contribution only from the fields vary in the fields vary in the fields
The fields vary in the fields vary in the fields vary in the bulk of the fields vary in the bulk of the bulk o

It became obvious that \varGamma is a total derivative of some vector current \varSigma_μ . Indeed, simple algebraic computation gives Γ = ε*µ*νλρσ Tr *^Gµ*ν*G*λρ*,*^σ = ∂*µ*Σ*µ*, where = 2
= 2 *x x* + *b* 2*t* + *b* 2*t* + *b* 2*t* ∂*M*⁵ Tr*(G*λρ*,*^σ δ*A*^ν + *^G*νλδ*A*ρσ *)d*σ*^µ* = ⁰*.* $\frac{1}{2}$ ous that Γ *d* dural delivative of some ved
d

Indeed, simple algebraic computation gives $\Gamma = \varepsilon_{\mu\nu\lambda\rho\sigma}$ Tr $G_{\mu\nu}G_{\lambda\rho,\sigma} = \partial_\mu \Sigma_\mu$, where

$$
\Sigma_{\mu} = 2\varepsilon_{\mu\nu\lambda\rho\sigma} \operatorname{Tr}(A_{\nu}\partial_{\lambda}A_{\rho\sigma} - \partial_{\lambda}A_{\nu}A_{\rho\sigma} - 2igA_{\nu}A_{\lambda}A_{\rho\sigma}).
$$

After some rearrangement and taking into account the definition of the field strength tensors
vector current: $\Gamma = c$ **Tr** *G* \cdot **A** vector current:

$$
\Sigma_{\mu} = \varepsilon_{\mu\nu\lambda\rho\sigma} \operatorname{Tr} G_{\nu\lambda} A_{\rho\sigma}.
$$

Tensor Gauge Theory and Mass Generation YM field strength tensor and in the rank-2 gauge field, picking up only its antisymmetric part. YM field strength tensor and in the rank-2 gauge field, picking up only its antisymmetric part. factor that we have at our disposal high-rank tensor gauge fields to build new invariants. The same is true for the \sim *Lensor Gauge Theory and Mass Generation*

Let us consider the fifth component of the vector current \mathcal{Z}_{μ} : *while the current Σ¹ and a five-dimensional manifold, we may restrict* the latter to one lower, we may restrict the latter to one lower, we may restrict the latter to one lower, we may restrict the latter to one lower fonsider the municomponent of the vector current z_{μ} : Let us consider the fifth component of the vector current Σ_{μ} : four-dimensional manifold. The restriction proceeds as follows. Let us consider the fifth component of the vector current Σ*µ*: $f^{\rm ac}$

$$
\Sigma \equiv \Sigma_4 = \varepsilon_{4\nu\lambda\rho\sigma} \operatorname{Tr} G_{\nu\lambda} A_{\rho\sigma}.
$$

Considering the component of the vector considering the vector consideration of the vector current of the const
The set of the component indices of four-dimensional space-time index and the sum over indices of four-dimensional space–time. Therefore we can reduce the can reduce this func Considering the fifth component of the vector current Z ≡ Z4 one can see that the vector current Z ≡ Z4 one can
The external the remaining indices will not repeat the external the external the external the external the ex the sum is restricted to the sum over indices of four-dimensional space–time. the *give* is restricted to the sum even indices of four dimensional speece 68 *G. Savvidy / Physics Letters B 694 (2010) 65–73* ϵ suili is restricted to

 T *his* i integral over four-dimensional space–time⁶: ⁵ The trace of the commutators vanishes: Tr*(*[*Aµ*; *^G*λρ*,*^σ δ*A*^ν]+[*A*λ; *^Gµ*^ν δ*A*ρσ]*)* = 0. This is the case when the gauge fields are independent on the fifth coordinate x_4 . four-dimensional space–time and, as we shall see, it is also gauge invariant up to the total divergence term. Therefore we shall consider four-dimensional space–time and, as we shall see, it is also gauge invariant up to the total divergence term. Therefore we shall consider e *gral* over four-dime *Ai*∂ *^j Ak* − *ig* .
م *Ai A ^j Ak*

*^d*4*^x* Tr *^G*νλ *^A*ρσ *.* (14)

$$
\int_{M_4} d^4x \Sigma = \varepsilon_{\nu\lambda\rho\sigma} \int_{M_4} d^4x \text{Tr } G_{\nu\lambda} A_{\rho\sigma}.
$$

As we claimed this functional is gauge invariant up to the total divergence term.

$$
\delta_{\xi} \int_{M_4} d^4x \Sigma = \varepsilon_{\nu\lambda\rho\sigma} \int_{M_4} \text{Tr}(-ig[G_{\nu\lambda}\xi]A_{\rho\sigma} + G_{\nu\lambda}(\nabla_{\rho}\xi_{\sigma} - ig[A_{\rho\sigma}\xi]) d^4x
$$

$$
= \varepsilon_{\nu\lambda\rho\sigma} \int_{M_4} \partial_{\rho} \text{Tr}(G_{\nu\lambda}\xi_{\sigma}) d^4x = \varepsilon_{\nu\lambda\rho\sigma} \int_{\partial M_4} \text{Tr}(G_{\nu\lambda}\xi_{\sigma}) d\sigma_{\rho} = 0.
$$

New gauge anomalies and topological invariants in various **dimensions** *dimensions* While the invariant Γ *(A)* and the vector current Σ*^l* are defined on a five-dimensional manifold, one can restrict the ant against *small* gauge transformations, but not under *large* opological invariants in vanous w dauge anomalies and topological inva defined in four dimensions and is going in the infinitesimal gauge transformations up to a total divergence transformations up to a total divergence transforma η in various to the corresponding one of η t_1 , t_2 , t_1 , t_2 , t_3 , t_5 and t_6 w asuge anomalies and topological in defined in four dimensions and is gauge invariant under in $rion to$ in various the Chern-Simons integral control control integrated with $\frac{1}{2}$ lew gauge anomalies and topological invariants in various defined in four dimensions and is gauge invariant under infinitesimal gauge transformations up to a total divergence

− Σ*(A)*

d

 $\sqrt{2}$

 $\overline{}$

Considering integral over four-dimens $slens \sim$ nsidering integral over four-dimensional space–time² Considering integral over four-dimensional space–time²

$$
\Sigma(A) = \frac{1}{32\pi^2} \int_{M_4} d^4x \,\varepsilon^{\nu\lambda\rho\sigma} \operatorname{Tr}(G_{\nu\lambda}A_{\rho\sigma}).
$$

This entity is an analog of the Chern–Simons integral³ \overline{a} $\sum_{n=1}^{\infty}$ for $\sum_{n=1}^{\infty}$ for the infinitesimal gauge transformations $\sum_{n=1}^{\infty}$ for $\sum_{n=1}^{\$ This entity is an analog of the Chern–Simons integral³ This entity is an analog of the Chern–Simons integral

$$
W(A) = \frac{g^2}{8\pi^2} \int_{M_3} d^3x \, \varepsilon^{ijk} \operatorname{Tr}\left(A_i \partial_j A_k - ig \frac{2}{3} A_i A_j A_k\right),
$$

but, *importantly, instead of being defined in three dimen*sions it is defined in four dimension tensor gauge fields allow to build a the Chern-Simons characteristic *us*, *we l Folls it is defined in four dimensions*. Thus, the non-Abelian $\overline{111}$ $\frac{1}{\alpha}$ $\ddot{ }$ but, *importantly, instead of being defined in three dimen*tensor gauge fields allow to build a natural generalization of the control of the contr $time$ sions it is defined in four dimensions. Thus, the non-Abelian tensor gauge fields allow to build a natural generalization of the Chern–Simons characteristic in four-dimensional space– time. time.

*d*4*x* ενλρσ Tr*(G*νλ*A*ρσ *).* (6) gauge transformations up to a to *manitesim* The functional $\Sigma(A)$ is invariant under infinitesimal λ gauge transformations up to a total divergence term. its gauge variation under \sim

The expression (9) is analogous to the corresponding one of $\mathcal{O}(n)$

and the density (10) to the density (10) to the non-Abelian anomaly in two-

Tensor Gauge Theory and Mass Generation T_{Ω} pear Cauga Theary and Maga Caparation refrom adage then y and mass acheration If the YM field strength *G*νλ vanishes, then the vector potential is equal to the pure gauge connection *A^µ* = *U*−∂*µU*. Inspecting the **ierisci cauge filedly and ivass ceneration Tensor Gauge Theory and Mass Generation** expression for the invariant Σ one can get convinced that it vanishes on such fields because there is a field strength tensor *G*νλ in the *n*_c *n*₀*G*, *adago n*₁*i*co = 0*.* (20) *n Ter* \mathbf{z} *Dr Gauge* 2 **Tensor Gauge Theory and Mass Generation** Foricer daage moory and mace denoration

In four dimensions the gauge fields have dimension of $[mass]^1$, $\frac{1}{\sqrt{2\pi}}$ In four dimensions the gauge folds have dimension of $Imaccl$ \mathcal{L} is may distinguish fields which are falling which are falling less faster at infinity and infinity and \mathcal{L} In four dime \mathbf{r}_{\bullet} *g a*^{*r*} dimensions the gauge fields have dimension of *[1* = 0*.* (20) In four dimensions the gauge fields

m and the control of the should introduce the mass parameter *m*: therefore if we intend to add this new density to the Lagrangian we *^m*Σ = *^m*ενλρσ Tr *^G*νλ *^A*ρσ *,* (17) In four dimensions the gauge fields have dimension of [*mass*] ! *n*uld in \overline{a} *Aa* λ*µ* − ∂λ∂*^µ Aa ss* pare \overline{a} *Aa neter m: A*
*AaU Ad Ad Ad Ad Ad Ad*_{*d*} *Ad*_{*d*} *Ad*_{*d*} *Ad*_{*d*} *Ad*_{*d*} *Ad* *Ad* *Ad* *Ad* $\overline{2}$ $\overline{\Omega}$ *Aa* d intr *Aa* lC − ∂ν ∂*^µ* $\overline{\mathsf{n}}$ *Aa* ss pare \mathbf{u} t − ∂λ∂*^µ* Lagratore if we Should mulbuuce the mass parameters

 $m\Delta = m\epsilon_{\nu\lambda\rho\sigma}$ if $G_{\nu\lambda}A_{\rho\sigma}$, $\ln 2 - \ln c \gamma_{\Lambda} \rho_0$ is $\sigma \gamma_{\Lambda} \rho_0$, $m\Sigma = m\varepsilon_{\nu\lambda\rho\sigma}$ Tr $G_{\nu\lambda}A_{\rho\sigma}$, 2 $m\angle = m$ μ *c* μ _{λ} ρ σ , σ , σ *m* $\overline{}$ $E = m \varepsilon_{\nu\lambda\rho\sigma} \operatorname{Tr} G_{\nu\lambda} A_{\rho\sigma}$

alternative mechanism for mass generation in gauge field theories in $\mathcal A$ where parameter m has units $[mass]^T$. where parameter m has units $[mass]^1$. hints at the fact that the theory turns out to be a massive theory. We shall see that the YM vector boson becomes massive, suggesting an 1 . m hara naramatar m has units $[macc]$ antistic parameter *Bold and antisianismess* through the mass term of mass term where parameter m has units ${\lceil mass \rceil}^1$. Φ ^{*n*} **b**ere pair *u*
γραματερ μ , bac, μ nite, μ *acc*¹ 1

we arrive at the followipg grotom of equations: we arrive at the following system of equations: ϵ and the particle spectrum of the particle spectrum of the mass parameters ϵ his at the turns out turns out that the shall see that the YM vector boson becomes massive that the YM vector b we arrive at the following system of equations: antisymmetric part *B*νλ of the rank-2 gauge field *A*νλ interacts through the mass term, the symmetric part *A^S* $\frac{1}{2}$ The corresponding free equations (*g* = 0) are:

$$
\partial^2 A_{\nu} - \partial_{\nu} \partial_{\mu} A_{\mu} + m \varepsilon_{\nu \mu \lambda \rho} \partial_{\mu} B_{\lambda \rho} = 0,
$$

$$
\partial^2 B_{\nu \lambda} - \partial_{\nu} \partial_{\mu} B_{\mu \lambda} + \partial_{\lambda} \partial_{\mu} B_{\mu \nu} + \frac{2m}{3} \varepsilon_{\nu \lambda \mu \rho} \partial_{\mu} A_{\rho} = 0.
$$

δξ *A^µ* = ∂*µ*ξ − *ig*[*Aµ,* ξ]*,* δξ *Aµ*^ν = −*ig*[*Aµ*^ν *,* ξ]*,* Let us compare the formulas (2.16) and (2.17) suggested in [39,40] for the transformation of antisymmetric field with the gauge transforand symmetric part $\mathbf{p}_{\nu\lambda}$ or u isymmetric part $B_{\nu\lambda}$ of the rank-2 gauge field $A_{\nu\lambda}$ interacts through the mass term, the symmetric part $A_{\nu\lambda}^s$ completely decouples $\frac{1}{2}$ anticummetric part R , of the rapk 2 gauge fold Λ , interacts through the mass term, the symmetric part A^3 , completely decouples antisymmetric part $B_{\nu\lambda}$ of the rank-2 gauge field $A_{\nu\lambda}$ interacts through the mass term, the symmetric part $A_{\nu\lambda}^S$ completely decouples

Tensor Gauge Theory and Mass Generation [∂]² *^B*νλ [−] ∂ν [∂]*µBµ*^λ ⁺ ∂λ∂*µBµ*^ν ⁺ 2*m and Mass deneration* LOO Thoory and Mace Canaration μ *y e i i c*_{*i*} *ei e*^{*i*} *ei e*^{*i*} *ei e*^{*i*} *ei e*^{*i*} *e*^{*i*} *e*

$$
(-k^2 \eta_{\nu\mu} + k_{\nu}k_{\mu})e_{\mu} + im\varepsilon_{\nu\mu\lambda\rho}k_{\mu}b_{\lambda\rho} = 0,
$$

$$
(-k^2 \eta_{\nu\mu}\eta_{\lambda\rho} + k_{\nu}k_{\mu}\eta_{\lambda\rho} - \eta_{\nu\mu}k_{\lambda}k_{\mu})b_{\mu\rho} + i\frac{2m}{3}\varepsilon_{\nu\lambda\mu\rho}k_{\mu}e_{\rho} = 0.
$$

four pure gauge solutions
$$
e_{\mu} = k_{\mu}
$$
, $b_{\nu\lambda} = 0$;
\n $e_{\mu} = 0$, $b_{\nu\lambda} = k_{\nu}\xi_{\lambda} - k_{\lambda}\xi_{\nu}$.

three solutions representing propagating modes: $\frac{1}{\sqrt{2}}$ gauge field equations, the mass spectrum of the lower-rank-sp propagating modes: $\frac{1}{2}$ three solutions represe ati $\overline{}$ \overline{p} modes: ng mones.

$$
e_{\mu}^{(1)} = (0, 1, 0, 0), \t b_{\gamma\gamma}^{(1)} = \frac{1}{i} \frac{M}{\sqrt{k^2 + M^2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix},
$$

\n
$$
e_{\mu}^{(2)} = (0, 0, 1, 0), \t b_{\gamma\gamma}^{(2)} = -\frac{1}{i} \frac{M}{\sqrt{k^2 + M^2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},
$$

\n
$$
e_{\mu}^{(3)} = \begin{pmatrix} 0, 0, 0, \frac{M}{\sqrt{k^2 + M^2}} \end{pmatrix}, \t b_{\gamma\gamma}^{(3)} = \frac{1}{i} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.
$$

It is a genuine superposition of vector and tensor fields solutions (24). It is a genuine superposition of vector and tensor fields. Let us consider the above solutions will factorize, α It is a genuine superposition of vector and tensor fields. *(*1*) (*2*)* $\mathbf{A} = \mathbf{A} \mathbf{A}$ and $\mathbf{A} = \mathbf{A}$

Topological invariants in various dimensions Topological invariants in various dimensions finitesimal gauge transformations up to a total divergence dependent on the fifth coordinate *x*4. This density is well lopological invariants in various dimensions

− Σ*(A)*

d

 $\sqrt{2}$

 $\overline{}$

The expression (9) is analogous to the corresponding one of $\mathcal{O}(n)$

and the density (10) to the density (10) to the non-Abelian anomaly in two-

Considering integral over four-dimens $slens \sim$ nsidering integral over four-dimensional space–time² Considering integral over four-dimensional space–time²

$$
\Sigma(A) = \frac{1}{32\pi^2} \int_{M_4} d^4x \,\varepsilon^{\nu\lambda\rho\sigma} \operatorname{Tr}(G_{\nu\lambda}A_{\rho\sigma}).
$$

This entity is an analog of the Chern–Simons integral³ \overline{a} $\sum_{n=1}^{\infty}$ for $\sum_{n=1}^{\infty}$ for the infinitesimal gauge transformations $\sum_{n=1}^{\infty}$ for $\sum_{n=1}^{\$ This entity is an analog of the Chern–Simons integral³ This entity is an analog of the Chern–Simons integral

$$
W(A) = \frac{g^2}{8\pi^2} \int_{M_3} d^3x \, \varepsilon^{ijk} \operatorname{Tr}\left(A_i \partial_j A_k - ig \frac{2}{3} A_i A_j A_k\right),
$$

but, *importantly, instead of being defined in three dimen*sions it is defined in four dimension tensor gauge fields allow to build a the Chern-Simons characteristic *us*, *we l Folls it is defined in four dimensions*. Thus, the non-Abelian $\overline{111}$ $\frac{1}{\alpha}$ $\ddot{ }$ *d pace*− but, *importantly, instead of being defined in three dimen*tensor gauge fields allow to build a natural generalization of the control of the contr $time$ sions it is defined in four dimensions. Thus, the non-Abelian tensor gauge fields allow to build a natural generalization of the Chern–Simons characteristic in four-dimensional space– time. time.

*d*4*x* ενλρσ Tr*(G*νλ*A*ρσ *).* (6) gauge transformations up to a to *manitesim* **d**₃*x* $\frac{1}{2}$ **Francis** Tr The functional $\Sigma(A)$ is invariant under infinitesimal gauge transformations up to a total divergence term. its gauge variation under \sim

Large Gauge Transformations 1 anne Gauge Transformations

$$
A^U_\mu = U^- A_\mu U + \frac{\imath}{g} \underbrace{U^- \partial_\mu U}_{}
$$

\n
$$
A^U_{\mu\lambda} = U^- A_{\mu\lambda} U + U^- A_\mu U_\lambda - U^- U_\lambda U^- A_\mu U + \frac{\imath}{g} \underbrace{(U^- \partial_\mu U_\lambda - U^- U_\lambda U^- \partial_\mu U)}_{}
$$

 $\mathcal{A} \rightarrow \mathcal{A}$ is the tensor fields have no symmetries with respect to the first index $\mathcal{A} \rightarrow \mathcal{A}$. where U_{λ} is the second term in the expansion of the unitary matrix $\mathcal{U}(\Xi(x,e))$ over the vector variable: where \mathcal{O}_{λ} is the second term in the expansion of the unitary matrix $\mathcal{U}(\Xi(x,e))$ over the $\frac{1}{2}$, $\frac{1$ ture) vanish. The YM field-strength Guvernithe Strength Guvernithe Guvernithe vanishes when the vector potential is equal is equal in the vector potential is equal in the vector potential is equal in the vector potential i

$$
\mathcal{U}(x, e) = U(x) + U_{\mu}(x)e^{\mu} + \dots,
$$

\n
$$
\mathcal{U}^{-}(x, e) = U^{-}(x) - U^{-}(x)U_{\mu}(x)U^{-}(x)e^{\mu} + \dots
$$

\n
$$
U = 1 - igL_{a} \xi^{a}(x), \quad U_{\mu} = -igL_{a} \xi^{a}_{\mu}(x), \dots
$$

\n
$$
A_{\mu}^{flat} = \frac{i}{g} U^{-} \partial_{\mu} U,
$$

\n
$$
A_{\mu\lambda}^{flat} = \frac{i}{g} (U^{-} \partial_{\mu} U_{\lambda} - U^{-} U_{\lambda} U^{-} \partial_{\mu} U).
$$

Large Gauge Transformations $B_{\rm eff}$ is denoted by a boundary term which is denoted by a boundary term wh vanishes when the gauge parameter ξσ *(x)* tends to zero sufran Caugo Transformational ant against *small* gauge transformations, but not under *large* targe Gauge Transformations.

we have to find out how $\mathcal{Z}(A)$ defined on a five-dimensional manifold, one can restrict the We have to find out how $\mathbb Z$ transforms under large gauge transformations. The expres**sion we found has the form** \blacksquare : **have to find out how** *.* (9)

$$
\Sigma(A^U) - \Sigma(A)
$$

= $\frac{i}{32\pi^2 g} \int_{M_4} d^4 x \,\varepsilon^{\mu\nu\lambda\rho} \partial_\lambda \operatorname{Tr}(G_{\mu\nu}U_\rho U^-).$ (9)

The expression (9) is the energy building in functional to four dimensions, considering gauge fields in-the Chern–Simons integral [11, 13, 18–20, 41–43]: analogous to the corresponding one of *a* The expression (9) is analogous to the corresponding one of

$$
W(A^U) - W(A)
$$

= $\frac{1}{8\pi^2} \int_{M_3} d^3x \, \varepsilon^{ijk} \partial_i \operatorname{Tr}(\partial_j U U^- A_k)$
+ $\frac{1}{24\pi^2} \int_{M_3} d^3x \, \varepsilon^{ijk} \operatorname{Tr} (U^- \partial_i U U^- \partial_j U U^- \partial_k U),$

Topological invariants in Yang-Mills Theory

the divergence of the axial U(1) current $J^A_\mu = \psi \gamma_\mu \gamma_5 \psi$, in four dimensions it is given by the \leq

$$
\partial^{\mu} J_{\mu}^{A} = -\frac{1}{16\pi^{2}} \varepsilon^{\mu\nu\lambda\rho} \operatorname{Tr}(G_{\mu\nu} G_{\lambda\rho})
$$

=
$$
-\frac{1}{4\pi^{2}} \varepsilon^{\mu\nu\lambda\rho} \partial_{\mu} \operatorname{Tr}\left(A_{\nu} \partial_{\lambda} A_{\rho} - i\frac{2}{3} g A_{\nu} A_{\lambda} A_{\rho}\right).
$$

Similarly, the non-Abelian anomaly appears in the covariant divergence of the non-Abelian left $J_{\mu}^{aL} = \bar{\psi}_L \gamma_\mu \gamma_5 \sigma^a \psi_L$ or right $J_{\mu}^{aR} = \bar{\psi}_R \gamma_\mu \gamma_5 \sigma^a \psi_R$ handed currents, such as $S_{\rm{m}}$ indepty the non-Abelian apomaly appears in the covariant divert divergence the non-Abelian divergence of the non-abelian divergence of $J^{aR} = \bar{\psi}_P \gamma \gamma_{\rm F} \sigma^a \psi$, handed curre $\int \frac{d\mu}{\mu} = \frac{\varphi}{L} \int \frac{\mu}{5}$ *y* φ *R* or right $\partial_{\mu} = \frac{\varphi}{R} \int \frac{\mu}{5}$ *y* φ *R* hand

$$
D^{\mu}J_{\mu}^{aL} = -\frac{1}{24\pi^{2}} \varepsilon^{\mu\nu\lambda\rho} \partial_{\mu} Tr[\sigma^{a}(A_{\nu}\partial_{\lambda}A_{\rho} - i\frac{1}{2}gA_{\nu}A_{\lambda}A_{\rho})].
$$

Topological invariants in Yang-Mills Theory Eur. Phys. J. C (2012) 72:2140 Page 3 of 13

1 he divergence of the a *a A A A L A i A C i currer* \mathbf{t} $I^A = \sqrt{I^A}$ the divergence of the axial U(1) current $J^A_\mu = \psi \gamma_\mu \gamma_5 \psi$,
in four dimensions it is given by the and the divergence of the axial U(1) current $J_{\mu}^{A} = \psi$
in four dimensions it is given by the *a a i s i s i s i i s i i by the* **1** *gA*ν*A*λ*A*^ρ **1** he divergence of the $axial U(1) curr$ $\frac{1}{2}$ *g*_{*A*} \mathcal{L} ^{*A*} \mathcal{L} _{*A*} \mathcal{L} ^{*A*} \mathcal{L} _{*A*} \mathcal{L} $\sum_{i=1}^{n}$

$$
\partial^{\mu} J_{\mu}^{A} = -\frac{1}{16\pi^{2}} \varepsilon^{\mu\nu\lambda\rho} \operatorname{Tr}(G_{\mu\nu} G_{\lambda\rho})
$$

=
$$
-\frac{1}{4\pi^{2}} \varepsilon^{\mu\nu\lambda\rho} \partial_{\mu} \operatorname{Tr}\left(A_{\nu} \partial_{\lambda} A_{\rho} - i\frac{2}{3} g A_{\nu} A_{\lambda} A_{\rho}\right).
$$
(12)

the $U_A(1)$ anomaly is given by a $2n$ -form, the higherdimensional analog of Eq. (12):

$$
d * JA \propto Tr(Gn) = d\omega_{2n-1},
$$

 $\frac{1}{2}$ is a generalization of the C_{hern}–Simons formation of the C_{hern}–Simons formation of the Chern–Simons formation of the Chern–Simons formation of the Chern–Simons formation of the Chern–Simons for the Chern–Sim where ω_{2n-1} is a generalization of the Chern–Simons form to 2*n* − 1 dimensions [6, 13]: ' 1

$$
\omega_{2n-1}(A) = n \int_0^1 dt \operatorname{Tr}(AG_t^{n-1}).
$$

$$
A = -igA^{a}_{\mu}L_{a} dx^{\mu}, \text{ with } G_{t} = tG + (t^{2} - t)A^{2}.
$$

terparts [6–9, 11, 13, 14, 17]. In *D* = 2*n* dimensions, the *UA(*1*)* anomaly is given by a 2*n*-form, the higheration of the density δω⁵ ⁼ *^d*ω¹ $4 + 15$ in (15) and with $\frac{1}{2}$ in (15) and with $\frac{1}{2}$ ²*n*−² in Anomalies and Topological invariants in Yang-Mills Theory

gauge variation of the ω_{2n-1} :

$$
\delta\omega_{2n-1} = d\omega_{2n-2}^1,\tag{21}
$$

where the $(2n-2)$ -form has the following integral representation

$$
\omega_{2n-2}^1(\xi, A) = n(n-1) \int_0^1 dt (1-t) \operatorname{Str}(\xi d\bigl(A G_t^{n-2} \bigr) \bigr),
$$

where $\xi = \xi^a L_a$ is a scalar gauge parameter and Str denotes a symmetrized trace. In $\mathcal{D} = 2n - 2$ dimensions, the non-Abelian anomaly is given by this $(2n - 2)$ -form, the higherdimensional analoge

$$
D * J_{\xi}^{L,R} \propto \omega_{2n-2}^1(\xi, A).
$$

$\frac{1}{2}$ New gauge anomalies in various dimensions where ^ξ ⁼ ^ξ *aLa* is a scalar gauge parameter and Str denotes where *ala* is a scalar gauge parameter and stream stream and stream strea

a symmetrized trace. In *D* = 2*n* − 2 dimensions, the non-

Our aim is to generalize the above construction by defining invariant densities in higher dimensions $D = 2n + 3 = 5, 7, 9, 11, ...$: *A* $\frac{1}{2}$ $\frac{1}{2}$ Our aim is to generalize the above construction (1), (6), by defining invariant densities in higher dimennaraliza∩ \mathbf{I} de above construction

$$
\Gamma_{2n+3}(A)=\mathrm{Tr}(G^nG_3)=d\sigma_{2n+2},
$$

where we are using a shorthand notation for the 3-form field-strength tensor $G_3 = dA_2 + [A, A_2]$ of the rank-2 gauge field $A_2 = -igA_{\mu\nu}^a L_a dx^{\mu} \wedge dx^{\nu}$ and $G_{3t} = tG_3 +$ $(t^2 - t)[A, A_2]$. The $(2n + 2)$ -form σ_{2n+2} is where we are using a shorthand $\frac{1}{2}$ $\sum_{i=1}^{\infty} \sum_{i=1}^{\infty} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{j$

$$
\sigma_{2n+2}(A, A_2) = \int_0^1 dt \operatorname{Tr} (AG_t^{n-1} G_{3t} + \cdots + G_t^{n-1} A G_{3t} + G_t^n A_2).
$$

mensionality of this density is $[mass]^{n(n+2)}$, and it can be used as an addition to t functional functional Lagrangian density $\frac{1}{2}$ and $\frac{1}{2}$ in density $\frac{1}{2}$ is defined as $\frac{1}{2}$ in the \frac $H(x)$ considerative of this depoits is $\int_{-\infty}^{\infty} e^{-x^2} dx^{n+2} dx$ and it can be used as an addition $(2n+2)$ -dimensional Lagrangian density gauge parameter (86), the corresponding densities are: dimensionality of this density is $[mass]^{n(n+2)}$, and it can be used as an addition to the

$$
\frac{1}{F^{n^2-2}} \int_{M_{2n+2}} \sigma_{2n+2}(A, A_2),
$$

 $\overline{}$

⁵ *(*ξ1*, A)* ⁼ Tr"

where *F* is a dimensional coupling constant, very similar to *IOpological Invariants in various dimensiol Topological invariants in various dimensions* T onological inv δω2*n*−¹ ⁼ *^d*ω¹ ²*n*−2*,* (21)

We also found a second series of exact 6*n*-forms constructed only in terms of the 3-form gauge field-strength *G*3: t ruotod o

$$
\Delta_{6n} = \text{Tr}(G_3)^{2n} = d\pi_{6n-1},
$$

where for the $(6n - 1)$ -form one gets the following expression: where \sin is a scalar gauge parameter and \sin is a scalar gauge parameter and Str denotes an

$$
\pi_{6n-1}(A, A_2) = 2n \int_0^1 dt \operatorname{Tr}(A_2 G_{3t}^{2n-1}).
$$

These forms are defined in $D = 6n - 1 = 5, 11, 17, \dots$ dimensions.

 $\mathbf A$ s well understood in $\mathbf A$ Our next aim is to construct possible gauge anomalies σ_{2n}^1 and π_{6n-2}^1 which follow from the generalized density σ_{2n+2} and π_{6n-1} (20). These potential anomalies are defined through the relation analogous to (21) : \longrightarrow Our next aim is to construct possible gauge anomalies σ_{2i}^1 $2n+1$ and π_{6n-2}^1 which follow from the generalized densities \rightarrow Slide 24

$$
\delta \sigma_{2n+2} = d \sigma_{2n+1}^1, \qquad \delta \pi_{6n-1} = d \pi_{6n-2}^1.
$$

ew gauge anomalies and topological gauge parameter (86), the corresponding densities are: *New gauge anomalies a* non gauge anomalies a *New gauge anomalies (t*² [−] *t)*[*A,A*2]. The *(*2*ⁿ* ⁺ ²*)*-form ^σ2*n*+² is σ2*n*+² (17) and π6*n*−¹ (20). These potential anomalies are nd topological invariants in variot *n*
2 *del 13 lui 13*
2 del 20 ²*n*+1*,* δπ6*n*−¹ ⁼ *^d*π¹ *New gauge anomalies and topological invariants in various* The low-dimensional densities can be extracted directly *dimensions*

$$
\sigma_3^1(\xi_1, A) = \text{Tr}(\xi_1 G),
$$

\n
$$
\sigma_5^1(\xi_1, A) = \text{Tr}\left(\xi_1 d\left(A dA + \frac{1}{2}A^3\right)\right),
$$

where $\xi_1 = \xi^a_\mu L_a dx^\mu$ is a 1-form gauge parameter where $\xi_1 = \xi_\mu^a L_a$ l_{x} -1 δ ₃ δ $\sum_{i=1}^{n}$

d when the σauge transformation is perfor $\mathbf n$ *A*2*A*² −, the *and when the gauge transformation*. gauge parameter ξ , then ³ *(*ξ1*, A)* = Tr*(*ξ1*G),* **E**rformed by a sc and when the gauge transformation is performed by a scalar

gauge parameter
$$
\xi
$$
, then
\n
$$
\sigma_5^1(\xi, A, A_2) = \text{Tr}\left(\xi d \left(A d A_2 + A_2 d A + \frac{1}{2} A^2 A_2 - \frac{1}{2} A A_2 A + \frac{1}{2} A_2 A^2 \right) \right).
$$

ts descendant (8 \mathcal{T}_4 $\frac{1}{2}$ $\frac{\text{F}}{\text{F}}$ $\frac{\text{F}}{\text{F}}$ $\frac{\text{F}}{\text{F}}$ $\frac{\text{F}}{\text{F}}$ $\frac{\text{F}}{\text{F}}$ $\frac{\text{F}}{\text{F}}$ $\frac{\text{F}}{\text{F}}$ $\frac{\text{F}}{\text{F}}$ $\frac{\text{F}}{\text{F}}$ to decondant $(s_{\alpha}1 - s_{\alpha}2)$ $\frac{1}{2}$ accordination (50 gauge $\frac{1}{2}$ its descendant ($\delta \sigma_5^1$ = $\sigma(\bm{l})$ $d\sigma_4^2$

$$
\sigma_4^2(\xi, \eta, A) = \text{Tr}\big((d\xi \eta + \eta d\xi - \xi d\eta - d\eta \xi) dA_2\big)
$$

ing gauge We also found a second series of exact 6*n*-forms constructed only in terms of the 3-form gauge field-strength *G*3: sent a potential Schwinger term in the correspondsecond-rank gauge field **A**² when we perform the standard α when we perform the standard st fined in odd dimensions it may have contribute contribution to the contribution to the contribution to the contribution to the contribution of \mathcal{L} may represent a potential Schwinger term in the correspondgauge aigeora ing gauge algebra [8, 9].

In the next section we shall present a short introduc-

 \mathcal{N} infinitesimal gauge transformation \mathcal{N} is defined by it is defined by it is defined by it is defined by

New gauge anomalies - transgression

In conclusion let us compare the Pontryagin–Chern–Simons densities \mathcal{P}_{2n} , ω_{2n-1} and ω_{2n-2}^1 in YM gauge theory with the corresponding densities Γ_{2n+3} , σ_{2n+2} , σ_{2n+1}^1 and Δ_{6n} , π_{6n-1} , π_{6n-2}^1 in the extended YM theory. The new characteristic classes are local forms defined on the space–time manifold and constructed from the curvature 2-form G and 3-form *G*3: ∆6*ⁿ* = Tr*(G*3*)* ω_{2n-1} and ω_{2n-2}^1 in YM gauge theory
no densities Γ_{2n+2} σ_{2n+2} σ_1^1 and 2 -form G_2 . variation can also be found, yielding the potential and \mathcal{L} and \mathcal{L} and \mathcal{L} and \mathcal{L} and \mathcal{L} *... ...* where ξ *^a*

$$
\Gamma_{2n+3} = \text{Tr}(G^n G_3) = d\sigma_{2n+2},
$$

\n
$$
\Delta_{6n} = \text{Tr}(G_3)^{2n} = d\pi_{6n-1}.
$$

the existence of these potential anomalies is based on the fact that they fulfill Wess–Zumino consistency conditions. At the same time, these invariant densities constructed on
the grasse time memifold house their own independent value the space–time manifold have their own independent value since they suggest the existence of new invariants characterizing topological properties of a manhold. At the same time, these invariant densities constructed on the space–time manifold have their own independent value since they suggest the existence of new invariants characterizing topological properties of a manifold.

New Topological Field Theories generalizing the Chern-Simons quantum field theory [28, 29, 30, 31, 32, 33, 34, 35, 36, 37, IVEW TOPOIOGICAL IEIG TIFEOITES

$$
Z(M, M_2^i, C^j, R) = \int {\cal D}A {\cal D}A_2 e^{ik \int_M \sigma_{2n+2}(A, A_2)} \prod_{i,j} Tr_{R_i} e^{i \oint_{M_2^i} A_2} Tr_{R_j} e^{i \oint_{C^j} A},
$$

where σ_{2n+2} is defined in (1.6) and k is a parameter, or on three-dimensional manifolds

$$
Z(M, M_3^i, C^j, R) = \int {\cal D}A {\cal D}A_3 e^{ik \int_M \psi_{2n+3}(A, A_3)} \prod_{i,j} Tr_{R_i} e^{i \oint_{M_3^i} A_3} Tr_{R_j} e^{i \oint_{C^j} A}
$$

as well as on higher dimensional ones, ψ_{2n+3} is defined in (1.8). In particular, for the partition function $Z(M)$ in four dimensions we get

$$
Z(M) = \int \mathcal{D}A \mathcal{D}A_2 e^{ik \int_{M_4} \sigma_4} = \int \mathcal{D}A \mathcal{D}A_2 e^{ik \int_{M_4} Tr(GA_2)}
$$

and in the large k limit the contribution to the path integral is dominated from the points of stationary phase which are, in the given case, the flat connections

$$
G = dA + A^2 = 0, \qquad G_3 = dA_2 + [A, A_2] = 0.
$$

$$
\mathcal{P}_{2n} \;\Rightarrow\; \omega_{2n-1} \;\Rightarrow\; \omega^1_{2n-2}.
$$

Therefore we shall perform the following transgressions:

$$
\Phi_{2n+4} \Rightarrow \psi_{2n+3} \Rightarrow \psi_{2n+2}^1,
$$

\n
$$
\Xi_{2n+6} \Rightarrow \phi_{2n+5} \Rightarrow \phi_{2n+4}^1,
$$

\n
$$
\Upsilon_{2n+8} \Rightarrow \rho_{2n+7} \Rightarrow \rho_{2n+6}^1.
$$

Thank You !

Publications

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