Double space T-dualization of type II superstring with coordinate dependent RR field

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Outline of the talk

1. Motivation and choice of the background
2. T-duality
3. Model and double space as a tool
4. Implementation of the T-dualization procedure
5. Double space T-dualization
6. Concluding remarks

RR field is coordinate dependent

- **Speculations concerning a fermionic substructure of space-time**, *Lettere al Nuovo Cimento, vol. 34, n. 1, 1982* - the fermionic coordinates are fundamental ones, while bosonic coordinates are ”composites”.
- Conjecture - superstring with RR field linearly dependent on x^μ produce Poisson bracket of the fermionic coordinates proportional to x^μ.
- A new tool - generalized Buscher T-dualization procedure (applicable only on the weakly curved background).

Model and background fields

- We will use pure spinor model of type II superstring theory.
- The choice of the background fields must obey consistency conditions. We choose - all background fields are constant except RR field strength, $F^{\alpha\beta} = f^{\alpha\beta} + C^{\alpha\beta}{}_{\mu}x^{\mu}$. The choice is in accordance with the consistency conditions.
- Initial assumptions (not in a contradiction with consistency conditions) - $C^{\alpha\beta}{}_{\mu}$ is an infinitesimal antisymmetric tensor.

General observations

- Dualities connect superstring theories (potential path to M-theory).
- T-dualization procedure gives the relation between initial and T-dual coordinates - transformation laws.
- We will use transformation laws in **canonical form.**
- Initial theory is geometrical one, which means

$$\{x^\mu, x^\nu\} = 0\,, \quad \{\pi_\mu, \pi_\nu\} = 0\,, \quad \{x^\mu(\sigma), \pi_\nu(\bar{\sigma})\} = \delta^\mu{}_\nu \delta(\sigma - \bar{\sigma})\,.$$

- Constant background case is solved both

Standard Buscher procedure of T-dualization

- Global shift symmetry exists $x^a \to x^a + b^a$, where index a is subset of μ.
- We introduce gauge fields $v_\pm^a$ and covariant derivatives $D_\pm x^\mu \equiv \partial_\pm x^a + v_\pm^a$.
- Additional term in the action

$$S_{gauge}(y, v_\pm) = \frac{1}{2}\kappa \int_\Sigma d^2\xi \left(v_+^a \partial_- y_a - \partial_+ y_a v_-^a \right) ,$$

  where y_a is Lagrange multiplier. It makes $v_\pm^a$ to be unphysical degrees of freedom.
- On the equations of motion for y_a we get initial action, while, fixing x^a to zero, on the equations of motion for $v_\pm^a$ we get T-dual action.

Generalized T-dualization - invariant coordinates

- Buscher procedure works along directions on which background fields do not depend - isometry directions.
- In the case of coordinate dependence - there is an additional step, introduction of invariant coordinate in the form

$$x^a_{inv} \equiv \int d\xi^\alpha D_\alpha x^a = x^a(\xi) - x^a(\xi_0) + \Delta V^a \,,$$

  where ξ^α parametrize the world-sheet, while ΔV^a is defined as line integral of gauge field.
- Fixing the gauge $x^a_{inv} \to \Delta V^a$.

Action

- The action is of the form

$$S = k\int_\Sigma d^2\xi \,[\partial_+ x^\mu \Pi_{+\mu\nu}\partial_- x^\nu$$
$$+ \frac{1}{2}(\partial_+\bar\theta^\alpha + \partial_+ x^\mu \bar\Psi^\alpha_\mu)\left(F^{-1}(x)\right)_{\alpha\beta}(\partial_-\theta^\beta + \Psi^\beta_\nu \partial_- x^\nu)\Big] \,,$$

where

$$\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2}G_{\mu\nu}\,, \quad F^{\alpha\beta} = f^{\alpha\beta} + C^{\alpha\beta}{}_\mu x^\mu \,.$$

- Assumption - $C^{\alpha\beta}{}_\mu$ (C) is infinitesimal and antisymmetric in $\alpha \leftrightarrow \beta$. All calculations are done up to the linear term in C i.e. quadratic and higher terms are considered to be zero.

Pure spinor formalism

- RNS and GS formalisms, but there are some problems.
- For example, in the RNS formalism there is a problem of the RR field strength coupling, because in order to write RR vertex you have to break world sheet supersymmetry (Friedan, Martinec and Shenker,Nuclear Physics B271 (1986) 93-165).
- On the other hand you have to use light-cone gauge to quantize the superstring theory in GS formalism (hep-th/0112160).
- We choose pure spinor formalism (https://pirsa.org/c11030), that is similar to GS one but does not suffer from such "diseases". The full form of the action is given in arXiv:hep-th/0405072. We have chosen

Double space

- Target space is spanned by x^μ, while T-dual space by y_μ. Double coordinate are consrtucted as $Z^M = (x^\mu, y_\mu)^T$.
- $O(D,D)$ invariant metric Ω^{MN}

$$\Omega^{MN} = \begin{pmatrix} 0 & 1_D \\ 1_D & 0 \end{pmatrix} .$$

- Generalized metric $\mathcal{H}_{MN}$ contains background fields of initial and T-dual theory satisfying

$$\mathcal{H}^T \Omega \mathcal{H} = \Omega \,,$$

which means that $O(D,D)$ symmetry is incorporated into theory.

T-dual action

- After implementing generalized T-dualization procedure (arXiv:2307.02438), we obtained

$$
{}^{b}S = \frac{\kappa}{2}\int_{\Sigma} d^2\xi \Big[\frac{1}{2}\bar{\Theta}_{-}^{\mu\nu}\partial_{+}y_{\mu}\partial_{-}y_{\nu} + \partial_{+}\bar{\theta}^{\alpha b}F^{-1}(V^{(0)})_{\alpha\beta}\partial_{-}\theta^{\beta} + \partial_{+}y_{\mu}{}^{b}\bar{\Psi}^{\mu\alpha b}F^{-1}(V^{(0)})_{\alpha\beta}\partial_{-}\theta^{\beta} + \partial_{+}\bar{\theta}^{\alpha b}F^{-1}(V^{(0)})_{\alpha\beta}{}^{b}\Psi^{\nu\beta}\partial_{-}y_{\nu}\Big]. \tag{1}
$$

y_μ is a dual coordinate and V^0 represents following integral

$$
\Delta V^{(0)\rho} = \frac{1}{2}\int_P d\xi^{+}\breve{\Theta}_{-}^{\rho_1\rho}\left[\partial_{+}y_{\rho_1} - \partial_{+}\bar{\theta}^{\alpha}(f^{-1})_{\alpha\beta}\Psi^{\beta}_{\rho_1}\right] - \frac{1}{2}\int_P d\xi^{-}\breve{\Theta}_{-}^{\rho\rho_1}\left[\partial_{-}y_{\rho_1} + \bar{\Psi}^{\alpha}_{\rho_1}(f^{-1})_{\alpha\beta}\partial_{-}\theta^{\beta}\right]. \tag{2}
$$

Notation

- $\bar{\Theta}_-^{\mu\nu}$ is inverse of $\bar{\Pi}_{\pm\mu\nu} = \Pi_{\pm\mu\nu} + \frac{1}{2}\bar{\Psi}_\mu^\alpha \left(F^{-1}(x)\right)_{\alpha\beta} \Psi_\nu^\beta = \check{\Pi}_{\pm\mu\nu} - \frac{1}{2}\bar{\Psi}_\mu^\alpha (f^{-1})_{\alpha\alpha_1} C_\rho^{\alpha_1\beta_1} x^\rho (f^{-1})_{\beta_1\beta}\Psi_\nu^\beta$
- $\bar{\Theta}_\mp^{\mu\nu} = \check{\Theta}_\mp^{\mu\nu} + \frac{1}{2}\check{\Theta}_\mp^{\mu\mu_1}\bar{\Psi}_{\mu_1}^\alpha (f^{-1})_{\alpha\alpha_1} C_\rho^{\alpha_1\beta_1} V^{(0)\rho}(f^{-1})_{\beta_1\beta}\Psi_{\nu_1}^\beta \check{\Theta}_\mp^{\nu_1\nu}$
  $\check{\Theta}_\mp^{\mu\nu} = \Theta_\mp^{\mu\nu} - \frac{1}{2}\Theta_\mp^{\mu\mu_1}\bar{\Psi}_{\mu_1}^\alpha (\bar{f}^{-1})_{\alpha\beta}\Psi_{\nu_1}^\beta \Theta_\mp^{\nu_1\nu}$
  $\bar{f}^{\alpha\beta} = f^{\alpha\beta} + \frac{1}{2}\Psi_\mu^\alpha \Theta_-^{\mu\nu}\bar{\Psi}_\nu^\beta$
  $\Theta_\mp^{\mu\nu} = -4(G_E^{-1}\Pi_\mp G^{-1})^{\mu\nu}\,, G_{E\mu\nu} = G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}$
  ${}^bF^{-1}(V^{(0)})_{\alpha\beta} =$
  $F^{-1}(V^{(0)})_{\alpha\beta} - \frac{1}{2}F^{-1}(V^{(0)})_{\alpha\alpha_1}\Psi_\mu^{\alpha_1}\bar{\Theta}_-^{\mu\nu}\bar{\Psi}_\nu^{\beta_1} F^{-1}(V^{(0)})_{\beta_1\beta}$
  ${}^b\bar{\Psi}^{\mu\alpha} = \frac{1}{2}\Theta_-^{\mu\nu}\bar{\Psi}_\mu^\alpha, \qquad {}^b\Psi^{\nu\beta} = -\frac{1}{2}\Psi_\mu^\beta\Theta_-^{\mu\nu}$

T-dual transformation laws

- $\bar{\Pi}_{+\mu\nu}\partial_- x^\nu = -\frac{1}{2}\partial_- y_\mu - \frac{1}{2}\bar{\Psi}^\alpha_\mu \left(F^{-1}(x)\right)_{\alpha\beta}\partial_-\theta^\beta - \beta^+_\mu(x)$
  $\bar{\Pi}_{+\mu\nu}\partial_+ x^\nu = \frac{1}{2}\partial_+ y_\nu - \frac{1}{2}\partial_+\bar{\theta}^\alpha \left(F^{-1}(x)\right)_{\alpha\beta}\Psi^\beta_\nu - \beta^-_\nu(x)$
  where $\beta^+_\mu(x) =$
  $-\frac{1}{2}(\bar{\theta}^\alpha + x^{\nu_1}\bar{\Psi}^\alpha_{\nu_1})(f^{-1})_{\alpha\alpha_1} C^{\alpha_1\beta_1}_\mu (f^{-1})_{\beta_1\beta}(\partial_-\theta^\beta + \partial_- x^{\nu_2}\Psi^\beta_{\nu_2})$
  $\beta^-_\mu(x) =$
  $-\frac{1}{2}(\partial_+\bar{\theta}^\alpha + \partial_+ x^{\nu_1}\bar{\Psi}^\alpha_{\nu_1})(f^{-1})_{\alpha\alpha_1} C^{\alpha_1\beta_1}_\mu (f^{-1})_{\beta_1\beta}(\theta^\beta + x^{\nu_2}\Psi^\beta_{\nu_2})$

Appropriate notation

- $\Psi^{\alpha}_{\mu} = \Psi^{\alpha}_{+\mu}\,, \bar{\Psi}^{\alpha}_{\mu} = \Psi^{\alpha}_{-\mu}\,, \theta^{\alpha} = \theta^{\alpha}_{+}\,, \bar{\theta}^{\alpha} = \theta^{\alpha}_{-}$
- $\left(F^{-1}(x)\right)_{\alpha\beta} = \Big(F^{-1}_{+}(x)\Big)_{\alpha\beta}\,, \quad \left(F^{-1}(x)\right)_{\beta\alpha} = \Big(F^{-1}_{-}(x)\Big)_{\alpha\beta}\,,$
- $f^{-1}C_{\mu}f^{-1} = C_{+\mu}\,, \quad f^{-1}C^{T}_{\mu}f^{-1} = C_{-\mu}$
- $\widehat{\Pi}_{\pm\mu\nu} = \bar{\Pi}_{\pm\mu\nu} - \frac{1}{2}(\theta^{\alpha}_{\mp} + x^{\nu_1}\Psi^{\alpha}_{\mp\nu_1})C_{\pm\mu\alpha\beta}\Psi^{\beta}_{\pm\nu}$
- $\widehat{\Theta}^{\nu\mu}_{\mp} = \bar{\Theta}^{\nu\mu_1}_{\mp}\left[\delta^{\mu}_{\mu_1} + \frac{1}{2}(\theta^{\alpha}_{\mp} + V^{(0)\nu_1}\Psi^{\alpha}_{\mp\nu_1})C_{\pm\mu_1\alpha\beta}\Psi^{\beta}_{\pm\nu_2}\breve{\Theta}^{\nu_2\mu}_{\mp}\right]$
- $\widehat{\Pi}_{\pm\mu\nu} = \widehat{B}_{\mu\nu} \pm \frac{1}{2}\widehat{G}_{\mu\nu}\,, \widehat{\Theta}^{\mu\nu}_{\mp} = -4(\widehat{G}^{-1}_{E}\widehat{\Pi}_{\mp}\widehat{G}^{-1})^{\mu\nu}$

T-dual transformation laws in double space

- $\pm\Omega_{MN}\partial_\pm Z^N = \check{\mathcal{H}}_{MN}\partial_\pm Z^N + \check{J}_{\pm M}$

where generalized metric is given as

$$\check{\mathcal{H}}_{MN} = \begin{pmatrix} \widehat{G}_{E\mu\nu}(V) & -2\widehat{B}_{\mu\mu_1}(\widehat{G}^{-1})^{\mu_1\nu}(V) \\ 2(\widehat{G}^{-1})^{\mu\nu_1}\widehat{B}_{\nu_1\nu}(x) & (\widehat{G}^{-1})^{\mu\nu}(x) \end{pmatrix},$$

and double current is

$$\check{J}_{\pm M} = \begin{pmatrix} \frac{1}{2}\widehat{G}_{\mu\nu_1}\widehat{\Theta}_\pm^{\nu_1\nu}(V) \\ (\widehat{G}^{-1})^{\mu\nu}(x) \end{pmatrix} J_{\pm\nu}$$

$$J_{\pm\nu} = \left[\Psi^\alpha_{\pm\nu}\left(F_\mp^{-1}(x)\right)_{\alpha\beta} - (\theta^\alpha_\pm + x^{\nu_1}\Psi^\alpha_{\pm\nu_1})C_{\mp\nu\alpha\beta}\right]\partial_\pm\theta^\beta_\mp .$$

Realization of T-duality in double space

- $$ {}^bZ^M = T^M{}_N Z^N \,, T^M{}_N = \begin{pmatrix} 0 & 1_D \\ 1_D & 0 \end{pmatrix} $$
- T-dual coordinates must have the T-dual transformation laws of the same form

$$ \pm\Omega_{MN}\partial_\pm {}^bZ^N = {}^b\check{\mathcal{H}}_{MN}\partial_\pm {}^bZ^N + {}^bJ_{\pm M}, $$

and it follows

$$ {}^b\check{\mathcal{H}}_{MN} = T_M{}^P \check{\mathcal{H}}_{PQ} T^Q{}_N, \quad {}^b\check{J}_{\pm M} = T_M{}^N \check{J}_{\pm N}. $$

Result

- From the first equation, after some calculations, we gett

$$ {}^{b}\widehat{\Pi}_{\pm\mu\nu}(x) = {}^{b}\widehat{B}_{\mu\nu}(x) \pm \frac{1}{2}{}^{b}\widehat{G}_{\mu\nu}(x) = \frac{1}{4}\widehat{\Theta}_{\mp\mu\nu} $$

  which is equal to the result obtained by analytical method.
- Comparing other components and using the second equation, we proved that in this case (coordinate dependent RR field) analytical and double space approach give the same result.

Summary

- We have analyzed T-dualization of the the type II superstring with all background fields constant except RR field strength that is linearly coordinate dependent.
- We proved equivalence of the analytical and double space (matrix) formalism.
- Noncommutativity and nonassociativity (exceeds the subject of the lecture) - even after fermionic T-dualization, Poisson bracket of the fermionic coordinates is equal to zero. We did not prove the conjecture.
- More details in arXiv:2307.02438 (B. Nikolic, D. Obric, Phys. Rev. D 109, 106004).